Revista Integración
Escuela de Matemáticas
Universidad Industrial de Santander
Vol. 25, No. 1, 2007, pág. 77–83

Non-Gaussianity and Loop Corrections in a Quadratic Two-Field Slow-Roll Model of Inflation. Part I

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Abstract. In this paper it is shown that it is possible to attain a high level of non-gaussianity in a particular quadratic two-field slow-roll model of inflation by taking care of loop corrections both in the spectrum and the bispectrum. A big f_{NL} is obtained even if ζ is generated during inflation. Five issues are in consideration when constraining the available parameter space: 1. we must be sure that we are in a perturbative regime; 2. we must apply the correct condition about the (possible) loop dominance in B_ζ or P_ζ; 3. we must satisfy the spectrum normalisation condition; 4. we must satisfy the spectral tilt constraint; 5. we must have enough inflation to solve the horizon problem.

Resumen. En el artículo se muestra que es posible lograr un alto nivel de no gaussianidad en un modelo particular de inflación de dos campos y del tipo slow-roll tomando en cuenta las correcciones de lazo tanto en el espectro como en el biespectro. Un valor grande para f_{NL} se obtiene incluso si ζ es genera durante la inflación. Cinco aspectos se tienen en cuenta al momento de restringir la ventana de parámetros disponible: 1. debemos asegurarnos de estar en un régimen perturbativo; 2. debemos aplicar la condición correcta acerca del (posible) dominio de las correcciones de lazo en B_ζ o P_ζ; 3. debemos satisfacer la condición de normalización del espectro; 4. debemos satisfacer la restricción sobre el índice espectral; 5. debemos tener suficiente inflación a fin de resolver el problema de horizonte.

Keywords: non-gaussianity, bispectrum, loop corrections.
***Palabras y frases claves*:** no gaussianidad, biespectro, correcciones de lazo.
***PACS*:** 98.80.Cq.

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1. The Model

Our quadratic two-field slow-roll model is described by the potential

$$V = V_0 \left(1 + \frac{1}{2}\eta_\phi \frac{\phi^2}{m_P^2} + \frac{1}{2}\eta_\sigma \frac{\sigma^2}{m_P^2}\right), \tag{1}$$

where the first term dominates and η_ϕ and η_σ are the usual η slow-roll parameters associated to the fields ϕ and σ. We are focused on the concave downward potential (η_ϕ and $\eta_\sigma < 0$) and for simplicity on the $\sigma = 0$ trajectory.

Since we are considering a slow-roll regime, the evolution of the fields in such a case is given by $\phi(N) = \phi_\star \exp(-N\eta_\phi)$ and $\sigma(N) = \sigma_\star \exp(-N\eta_\sigma)$, in terms of the field values $\phi_\star$ and $\sigma_\star$ at the time when the relevant cosmological scales exit the horizon. Such expressions, together with Eq. (1), seed the δN formalism in order to calculate the spectrum and the bispectrum of the curvature perturbation including the tree-level and the one-loop contributions [1, 2, 3]:

$$\mathcal{P}_\zeta^{tree} = \frac{1}{\eta_\phi^2 \phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2, \quad \mathcal{P}_\zeta^{1-loop} \simeq \frac{\eta_\sigma^2}{\eta_\phi^4 \phi_\star^4} \exp[4N(|\eta_\sigma| - |\eta_\phi|)] \left(\frac{H_\star}{2\pi}\right)^4 \ln(kL), \tag{2}$$

$$\begin{aligned} B_\zeta^{tree} &= -\frac{1}{\eta_\phi^3 \phi_\star^4}\left(\frac{H_\star}{2\pi}\right)^4 4\pi^4 \left(\frac{\sum_i k_i^3}{\prod_i k_i^3}\right), \\ B_\zeta^{1-loop} &\simeq \frac{\eta_\sigma^3}{\eta_\phi^6 \phi_\star^6} \exp[6N(|\eta_\sigma| - |\eta_\phi|)] \left(\frac{H_\star}{2\pi}\right)^6 \ln(kL) 4\pi^4 \left(\frac{\sum_i k_i^3}{\prod_i k_i^3}\right), \end{aligned} \tag{3}$$

where L is the infrared cutoff chosen so that the quantities are calculated in a minimal box.

The important factor in the loop corrections is the exponential. This exponential function is directly related to the quadratic form of the potential with a leading constant term. It will give a large contribution if $|\eta_\sigma| > |\eta_\phi|$. I have chosen the concave downward potential in order to satisfy the spectral tilt constraint, which makes either $\eta_\phi < 0$, if $\mathcal{P}_\zeta \simeq \mathcal{P}_\zeta^{tree}$, or $\eta_\sigma < 0$, if $\mathcal{P}_\zeta \simeq \mathcal{P}_\zeta^{1-loop}$, while keeping $|\eta_\sigma| > |\eta_\phi|$.

2. Constraints to Have a Reliable Parameter Space

2.1. Working in a Perturbative Regime

It has been proved by means of an analysis of the Feynman-like diagrams [2, 3], and also by an alternative non-perturbative approach [4], that there exists what it is called a "coupling constant" g for the potential in Eq. (1). Such a coupling constant allows us to work in a perturbive regime if it is much less than one:

$$g = \frac{\eta_\sigma^2}{\eta_\phi^2}\frac{1}{\phi_*^2}\left(\frac{H_*}{2\pi}\right)^2 \exp[2N(|\eta_\sigma| - |\eta_\phi|)] \ll 1$$

$$\Rightarrow \quad \left(\frac{\phi_*}{m_P}\right)^2 \gg \frac{r\mathcal{P}_\zeta}{8}\frac{\eta_\sigma^2}{\eta_\phi^2}\exp[2N(|\eta_\sigma| - |\eta_\phi|)]\,, \qquad (4)$$

being r the tensor to scalar ratio.

2.2. Tree-Level or Loop Dominance

Because of the exponential factors in Eqs. (2) and (3) it might be possible that the loop corrections dominate over $\mathcal{P}_\zeta$ and/or B_ζ. There are three posibilities in complete connection with the position of the ϕ field when the relevant scales are exiting the horizon. Here I will consider only the intermediate $\phi_\star$ region, corresponding to the case when B_ζ is dominated by one-loop corrections and $\mathcal{P}_\zeta$ is dominated by the tree-level term, because this is the only possibility which gives interesting and observationally relevant results.

B_ζ Dominated by One-Loop Corrections and $\mathcal{P}_\zeta$ Dominated by the Tree-Level Term: the Intermediate $\phi_\star$ Region

Looking at Eqs. (2) and (3) it is required in this case that

$$\frac{\eta_\sigma^2}{\eta_\phi^2}\exp[4N(|\eta_\sigma| - |\eta_\phi|)] \ll \frac{1}{\frac{1}{\phi_\star^2}\left(\frac{H_*}{2\pi}\right)^2}\,, \qquad \frac{\eta_\sigma^3}{\eta_\phi^3}\exp[6N(|\eta_\sigma| - |\eta_\phi|)] \gg \frac{1}{\frac{1}{\phi_\star^2}\left(\frac{H_*}{2\pi}\right)^2}\,, \qquad (5)$$

which combines to give

$$\frac{r\mathcal{P}_\zeta}{8}\frac{\eta_\sigma^2}{\eta_\phi^2}\exp[4N(|\eta_\sigma| - |\eta_\phi|)] \ll \left(\frac{\phi_\star}{m_P}\right)^2 \ll \frac{r\mathcal{P}_\zeta}{8}\frac{\eta_\sigma^3}{\eta_\phi^3}\exp[6N(|\eta_\sigma| - |\eta_\phi|)]\,. \qquad (6)$$

2.3. Spectrum Normalisation Condition

Since we are considering ζ being generated during inflation, we must satisfy the appropiate spectrum normalisation condition. According to Eq. (2) if $\mathcal{P}_\zeta$ is dominated by the tree-level term, we have

$$\mathcal{P}_\zeta^{tree} = \frac{1}{\eta_\phi^2\phi_\star^2}\left(\frac{H_\star}{2\pi}\right)^2 = \frac{1}{\eta_\phi^2}\left(\frac{m_P}{\phi_*}\right)^2\frac{r\mathcal{P}_\zeta}{8} = \mathcal{P}\,, \qquad (7)$$

which reduces to

$$\left(\frac{\phi_\star}{m_P}\right)^2 = \frac{1}{\eta_\phi^2}\frac{r}{8}\,. \tag{8}$$

Notice that in such a situation, the value of the ϕ field when the relevant scales are exiting the horizon depends exclusively on the tensor to scalar ratio, once η_ϕ has been fixed by the spectral tilt constraint.

2.4. Spectral Tilt Constraint

The current WMAP value for the spectral tilt is $n_\zeta - 1 = -0.042 \pm 0.016$, and again we will consider only the case when $\mathcal{P}_\zeta$ is dominated by the tree-level term. That means that the usual spectral index formula applies,

$$n_\zeta - 1 = -2\epsilon - 2m_P^2 \frac{\sum_{ij} V_i N_j N_{ij}}{V \sum_i N_i^2}\,, \tag{9}$$

giving the following result once the derivatives of N with respect to $\phi_\star$ and $\sigma_\star$ have been calculated:

$$n_\zeta - 1 = -2\epsilon + 2\eta_\phi\,. \tag{10}$$

The efect of the ϵ parameter may be discarded in the previous expression, since ϵ is much less than $|\eta_\phi|$:

$$\epsilon = \frac{m_P^2}{2}\frac{V_\phi^2 + V_\sigma^2}{V^2} = |\eta_\phi| \left[\frac{1}{2}|\eta_\phi| \left(\frac{\phi}{m_P}\right)^2\right] \ll |\eta_\phi|\,, \tag{11}$$

according to the prescription that the potential in Eq. (1) is dominated by the constant term. Thus, using the central value for $n_\zeta - 1$, we get

$$\eta_\phi = -0.021\,. \tag{12}$$

2.5. Amount of Inflation

It is well-known that the number of e-folds of expansion from the time the cosmological scales exited the horizon to the end of inflation must be around or less than 70. The slow-roll evolution of the ϕ field tells us that such an amount of inflation is given by

$$N = \frac{1}{|\eta_\phi|}\ln\left(\frac{\phi_{end}}{\phi_*}\right) \lesssim 70\,. \tag{13}$$

Up to the moment we do not have a definite mechanism to end inflation in this model. It could not be by means of the violation of the $\epsilon < 1$ condition, since this would imply extrapolating the results to a region where the potential in Eq. (1) is no longer dominated by the constant term, and that, of course, would spoil the large non-gaussianity generated. Therefore, it will be assumed that inflation comes to an end when $|\eta_\phi|\phi^2/2m_P^2 \sim 10^{-2}$ to be in a safe side, hoping that a mechanism to end inflation at this point may be implemented keeping, or perhaps enhancing, the generated non-gaussianity. Coming back to Eq. (13), we get

$$N = \frac{1}{|\eta_\phi|} \ln\left(\frac{0.141 m_P}{|\eta_\phi|^{1/2}\phi_*}\right) \lesssim 70\,, \tag{14}$$

which leads to

$$\frac{\phi_*}{m_P} \gtrsim \frac{0.141}{|\eta_\phi|^{1/2}} \exp(-70|\eta_\phi|)\,. \tag{15}$$

3. f_{NL}

In this section I will calculate the level of non-gaussianity represented in the parameter f_{NL} by taking into account the constraints presented in Sections 1 and 2 [3]. The level of non-gaussianity, according to the expressions in Eqs. (2) and (3), is in this case given by

$$\begin{aligned}
-\frac{6}{5}f_{NL} =& \frac{B_\zeta^{1-loop}}{4\pi^4 \frac{\sum_i k_i^3}{\prod_i k_i^3}(\mathcal{P}_\zeta^{tree})^2} \simeq \frac{\eta_\sigma^3}{\eta_\phi^2\phi_\star^2}\exp[6N(|\eta_\sigma| - |\eta_\phi|)]\left(\frac{H_\star}{2\pi}\right)^2 \ln(kL) \\
=& \frac{\eta_\sigma^3}{\eta_\phi^2}\exp[6N(|\eta_\sigma| - |\eta_\phi|)]\left(\frac{m_P}{\phi_*}\right)^2 \frac{r\mathcal{P}_\zeta}{8}\ln(kL) \\
=& \eta_\sigma^3 \exp[6N(|\eta_\sigma| - |\eta_\phi|)]\mathcal{P}_\zeta \ln(kL)\,,
\end{aligned} \tag{16}$$

so that

$$-\frac{6}{5}f_{NL} \approx -2.5\times 10^{-9}|\eta_\sigma|^3 \exp[286\ \ln(5.78\times 10^{-2} r^{-1/2})\,(|\eta_\sigma| - 0.021)]\,, \tag{17}$$

where in the last line we have used expressions in Eqs. (8), (12), and (14).

In figure 1 we show lines of constant f_{NL} in the plot r vs $|\eta_\sigma|$ for the intermediate $\phi_\star$ region in agreement with the constraint in Eq. (6). Notice that by implementing the spectral tilt constraint in Eq. (12) to the spectrum normalisation constraint in Eq. (8) and the amount of inflation constraint in Eq. (15), we may conclude that the tensor to scalar ratio is bounded from below: $r \gtrsim 1.77\times 10^{-4}$.

The WMAP satellite observations provide an upper bound on the f_{NL} value: $|f_{NL}| \lesssim 120$, being one of main goals of the forthcoming observations of the anisotropies of the cosmic microwave background radiation to reduce even more such a bound. It is estimated that in the future WMAP will do it up to $|f_{NL}| \lesssim 20$, while the PLANCK satellite will do it up to $|f_{NL}| \lesssim 5$.

As it is seen in the plot, the PLANCK observationally allowed range of values for positive f_{NL}: $5 \lesssim f_{NL} \lesssim 120$ is completely inside the intermediate $\phi_\star$ region, as required. So we conclude that *if B_ζ is dominated by one-loop terms, but $\mathcal{P}_\zeta$ is dominated by the tree-level term, sizeable non-gaussianity is generated even if ζ is generated during inflation.* We also conclude that *for non-gaussianity to be observable, primordial gravitational waves must be undetectable.*

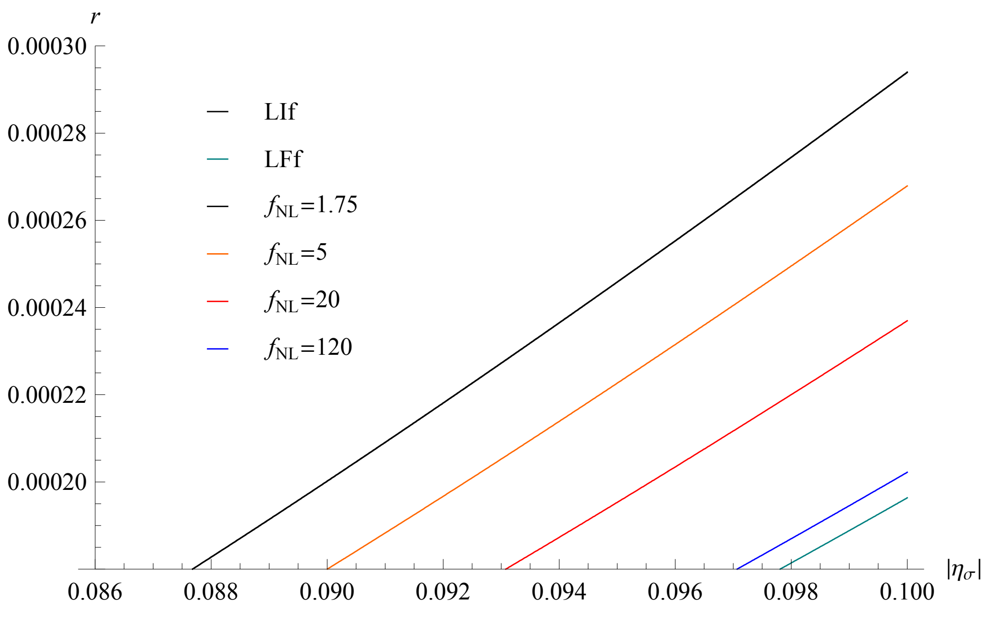

Figure 1. Lines of constant f_{NL} in the r vs $|\eta_\sigma|$ plot. The intermediate $\phi_\star$ region is delimited by the **LIf** and **LFf** lines. The PLANCK observationally allowed range of values for positive f_{NL}: $5 \lesssim f_{NL} \lesssim 120$ is completely inside the intermediate $\phi_\star$ region. Notice that the **LIf** line matches almost exactly the $f_{NL} = 1.75$ line.

Acknowledgments: This work is supported by COLCIENCIAS grant No. 1102-333-18674 CT-174-2006, and by DIEF (UIS) grant No. 5134. The author thanks the ECOS-NORD programme, grant No. C06P02, for its mobility support.

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