



Robust Estimation for a CSTR Using a High Order Sliding Mode Observer and an Observer-Based Estimator

Estimación Robusta para un CSTR Usando Observadores por Modos Deslizantes de Alto orden y un Estimador Basado en Observador

Estimação Robusta para um CSTR Usando uma Ordem Elevada Modo Deslizante Observador e um Estimador Baseado em Observador

Héctor A. Botero^{1*}; Esteban Jiménez-Rodríguez¹; Oscar Jaramillo¹; Juan Diego Sánchez Torres²

¹Universidad Nacional de Colombia, Sede Medellín, Grupo de Investigación en Procesos Dinámicos-Kalman
Crr 80 # 65-223. Medellín, Colombia

²Departamento de Matemáticas y Física, ITESO – Universidad Jesuita de Guadalajara, México.

*habotero@unal.edu.co

Fecha Recepción: 12 de junio de 2015

Fecha Aceptación: 12 de septiembre de 2016

Abstract

This paper presents an estimation structure for a continuous stirred-tank reactor, which is comprised of a sliding mode observer-based estimator coupled with a high-order sliding-mode observer. The whole scheme allows the robust estimation of the state and some parameters, specifically the concentration of the reactive mass, the heat of reaction and the global coefficient of heat transfer, by measuring the temperature inside the reactor and the temperature inside the jacket. In order to verify the results, the convergence proof of the proposed structure is done, and numerical simulations are presented with noiseless and noisy measurements, suggesting the applicability of the posed approach.

Keywords: *observers for chemical processes, robust observer design, sliding mode algorithms, uncertain systems.*

Resumen

Este trabajo presenta una estructura de estimación para un reactor de tanque agitado en continuo, la cual se compone de un estimador basado en observador por modos deslizantes acoplado con un observador por modos deslizantes de alto orden. Todo el esquema permite la estimación robusta del estado y algunos parámetros, concretamente la concentración de la masa reactiva, el calor de reacción y el coeficiente global de transferencia de calor, a partir de la medición de las temperaturas al interior del reactor y de la chaqueta. Para verificar los resultados, se realizan pruebas de convergencia de la estructura propuesta, y se presentan simulaciones numéricas con mediciones ruidosas y sin ruido, lo que sugiere la aplicabilidad del enfoque planteado.

Palabras clave: *observadores para procesos químicos, diseño de observadores robustos, algoritmos de modos deslizantes, sistemas con incertidumbre.*

Resumo

Este trabalho apresenta uma estrutura de estimativa para um reator de tanque agitado continuamente, que é composta por um estimador baseado em observador deslizante de modo acoplado com um alto ordem modo deslizante observador. Todo o esquema permite a estimativa robusta do estado e alguns parâmetros, a saber, a concentração da massa de reação, o calor da reação e o coeficiente de transferência total de calor, a partir da medição da temperatura no interior do reator e da jaqueta. Para verificar os resultados, as evidências de convergência da estrutura proposta são feitas, e simulações numéricas são apresentadas com medidas ruidosas e sem ruído, o que sugere a aplicação da abordagem proposta.

Palabras-chave: *observadores do processo químico, projeto observadores robustos, modos deslizantes algoritmos, sistemas com a incerteza.*

Introduction

For numerous applications, including chemical processes, it is well known that nonlinear state observers, parameter and unknown inputs estimators have been becoming of great interest. The main uses for those methods are the design of observer-based controllers, the synthesis of fault detection and isolation methods [1], among other applications. An important class of nonlinear observers is the Sliding Mode Observers (SMO) [2–4], which have the main features of the Sliding Mode (SM) algorithms. Those algorithms, are proposed with the idea to drive the dynamics of a system to a sliding manifold, that is an integral manifold with finite reaching time [5], exhibiting very interesting features such as work with reduced observation error dynamics, the possibility of obtain a step by step design, robustness and insensitivity under parameter variations and external disturbances, and finite-time stability [2,6]. In addition, the last feature can be extended to uniform finite time stability [7] also known as fixed-time stability [8], allowing the controllers and estimators to converge with a settling-time independent to the initial conditions. On the other hand, it is said usually that the sliding mode algorithms present two main disadvantages:

- i. they are usually assumed to be more sensitive to noise than linear controllers and estimators [9], and
- ii. the so-called chattering which is a high frequency oscillation due to the effect of discontinuities of the functions used to induce sliding modes (e.g. the sign function) on the unmodeled dynamics of the system [2].

However, for the disadvantage (i), using the steady state error as performance index, it is shown that, under the bounded disturbance hypothesis, linear and discontinuous algorithms are equally sensitive

to noise. Therefore, discontinuous algorithms are a better choice under both noise and disturbance presence conditions [10]. Besides, for the disadvantage (ii), several approaches have been proposed to reduce or avoid chattering. A first example is the use of continuous and smooth approximations of the sign function as linear saturation or sigmoid functions [11], with this solution only a quasi-sliding motion can be forced in a vicinity of the desired manifold, decreasing the performance and the robustness of the algorithm [12]. Another approach to induce sliding modes reducing chattering is to use continuous but non-differentiable functions or with discontinuous derivatives, instead of a discontinuous function. These methods are the so-called High Order Sliding Mode (HOSM) algorithms [13], extending the idea of the SM actuating on the time derivatives of the sliding manifold, preserving the main features of the original SM approach. In addition, for a SMO design case, the chattering reduces to a numerical problem [14]. Hence, some SMO have attractive properties similar to those of the Kalman filter but with simpler implementation [15]. For chemical processes, SM observers have been applied for several cases. Some examples are shown in [16–19]. The idea of coupling an observer with an Observer-Based Estimator (OBE) was used in [20] and, the idea of applying a High Order Sliding Mode Observer (HOSMO) for state and parameter estimation in a Continuous Stirred-Tank Reactor (CSTR), assuming the parameter to be estimated as an unknown input, was introduced by [21]. Here, measuring the temperature inside the reactor, the observer estimates the heat of reaction, and the concentration of the reactive. In addition, in order to facilitate the estimation procedure, the global heat transfer coefficient was assumed to be a known constant. No dynamics of the temperature inside the jacket were considered

in the mentioned approach. This paper proposes an extension of the last approaches, considering the global heat transfer coefficient as an unknown parameter to be estimated. This assumption conduces to a better approximation of the CSTR real operation conditions. As a first step, an OBE for the global heat transfer coefficient estimation is used. With this estimation, a HOSMO for state and input estimation [22] is used to estimate the heat of reaction, and the concentration of reactive. The heat of reaction is a parameter which is considered to be an unknown input for the observer design. The HOSMO is based on a real-time differentiator [23]. In the following, the next section presents the mathematical model of the CSTR. Then, the estimation structure will be described. The paper

continues with the results of numerical simulation. Finally, the conclusions of this paper are presented.

Mathematical Model for the CSTR

The CSTR considered performs an exothermic chemical reaction from the reactant Γ to the product $\Phi(\Gamma \rightarrow \Phi)$. This plant has also a recirculation flow in the jacket, which allows improving its controller design [24].

The state equations of the CSTR are presented in two subsystems as follows, with the outputs T and T_j :

$$\left\{ \begin{array}{l} \dot{T}_j = \frac{F_{jf}}{V_j} (T_{jf} - T_j) - \frac{\Lambda}{\rho_j C_{pj} V_j} (T_j - T) \end{array} \right. \quad (1)$$

$$\left\{ \begin{array}{l} \dot{T} = \frac{F}{V} (T_{in} - T) - \frac{\Delta H}{\rho C_p} k_0 C_A e^{-\frac{E}{RT}} + \frac{\Lambda}{\rho C_p V} (T_j - T) \\ \dot{C}_A = \frac{F}{V} (C_{in} - C_A) - k_0 C_A e^{-\frac{E}{RT}} \end{array} \right. \quad (2)$$

Here, F is the flow into the reactor, V is the volume of the reaction mass, C_{in} is the reactive input concentration, C_A is the concentration of the reactive inside the reactor, k_0 is the Arrhenius kinetic constant, E is the activation energy, R is the universal gas constant, T is the temperature inside the reactor, T_{in} is the inlet temperature of the reactant, ΔH is the heat of reaction (being an exothermic reaction, this corresponds to the released energy during the reaction), and in this article is considered an unknown input because it is an uncertain parameter, ρ is the density of the mixture in the reactor, C_p is the heat capacity of food, the parameter $\Lambda = UA$ is the global coefficient of heat transfer, where U is the overall coefficient of heat transfer and A is the heat transfer area, T_j is the temperature inside the jacket, F_{jf} is the feeding flow, V_j is the jacket volume, T_{jf} is the inlet temperature to the jacket, ρ_j is the density of jacket flow, and C_{pj} is the heat capacity of the jacket flow [24].

Estimation System Design

In this section the estimation system for the CSTR is proposed, it estimates the following variables:

- The concentration of the reactant inside the reactor, C_A (state variable), due to expensive sensors.

- The global coefficient of heat transfer, Λ (parameter), which depends on tank level (although in this paper it was considered constant), the stirring speed inside the reactor, the cleaning degree of the surface inside the reactor, and speed of the cooling flow inside the jacket; making its estimation a hard task [25].
- The heat of reaction, ΔH (parameter), which is uncertain due to experimental measurement complexity of thermal and kinetic phenomena that involve it [18].

Considering Subsystem (1), it can be noted that the global coefficient of heat transfer, Λ , is the only unknown parameter.

Taking advantage of this characteristic, an estimation system based on cascade connection of an OBE with a HOSMO is proposed as follows:

- Using measurements of T , T_j and, an OBE based on Subsystem (1), an estimation of the parameter Λ , ($\hat{\Lambda}$), is obtained.
- Using measurements of T , T_j , the estimated parameter $\hat{\Lambda}$ and, a HOSMO based on Subsystem (2), estimations of the state variable C_A , (\hat{C}_A), and the parameter ΔH , ($\hat{\Delta H}$) are obtained.

For the last case, the HOSMO structure allows to consider the parameter ΔH as an unknown input.

With this assumption, the estimation system is robust against variations of ΔH .

The whole structure for state and parameter estimation of CSTR is shown in Figure 1.

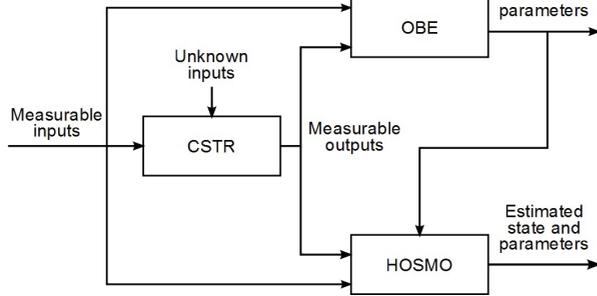


Figure 1. Structure for state and parameter estimation of CSTR.

Regarding the cascade structure of the estimation system, it will be started with the parameter Λ estimation.

At first, it must be noted that the HOSMO allows the joint estimation of state and parameters, considering the last ones as unknown inputs. However, it is not possible to fulfill the requirement by the HOSM in order to estimate simultaneously the parameters Λ and ΔH .

Therefore, the use of an OBE to estimate Λ is proposed here. Initially, it is highlighted that the proposed scheme, in addition to the temperature measurement T_j , also requires an estimation of its time derivative \dot{T}_j .

$$\begin{cases} \dot{\hat{T}}_j = \frac{F_{jf}}{V_j} (T_{jf} - \hat{T}_j) - \frac{\hat{\Lambda}}{\rho_j C_{pj} V_j} (T_j - z_0) + k_1 \phi_1(\tilde{T}_j) \\ \dot{\hat{\Lambda}} = k_2 \frac{\rho_j C_{pj} V_j}{\delta_j} \phi_2(\tilde{T}_j) \end{cases} \quad (4)$$

where $\tilde{T}_j = T_j - \hat{T}_j$. The OBE input injections are given by $\phi_1(\tilde{T}_j) = |\tilde{T}_j|^{\frac{1}{2}} + \mu |\tilde{T}_j|^{\frac{3}{2}}$ and $\phi_2(\tilde{T}_j) = \frac{1}{2} |\tilde{T}_j|^0 + 2\mu \tilde{T}_j + \frac{3}{2} \mu^2 |\tilde{T}_j|^2$ with $\mu \geq 0$. Finally, k_1 and k_2 are the OBE gains. The gain δ_j is a positive constant which takes the value of a lower bound for $|z_0 - T_j|$, the existence of this parameter is verified taking into account that all the temperatures are measured in Kelvin degrees and the condition $z_0 > T_j$ for all the time.

Note that the system (4) can be considered as a second order sliding mode extension of the linear OBE presented in [26].

This estimation is performed by using a robust exact sliding mode differentiator [23], with the following structure:

$$\begin{aligned} \dot{z}_0 &= -\lambda_0 L^{\frac{1}{3}} [z_0 - T]^{\frac{2}{3}} + z_1 \\ \dot{z}_1 &= -\lambda_1 L^{\frac{1}{2}} [z_1 - \dot{z}_0]^{\frac{1}{2}} + z_2 \\ \dot{z}_2 &= -\lambda_2 L [z_2 - \dot{z}_1]^0 \end{aligned} \quad (3)$$

where it is defined the function $|\cdot|^\alpha = |\cdot|^\alpha \text{sign}(\cdot)$ for $\alpha \geq 0$; z_0 is the estimation of T , z_1 is the estimation of its time derivative \dot{T} and, z_2 is the estimation of \ddot{T} , with $\lambda_0, \lambda_1, \lambda_2$ being positive gains. Let the third derivative $T^{(3)}$ be bounded as $|T^{(3)}| < \delta T$ for all the time with δT a positive constant, therefore, the gain L must be such that $L > \delta T$.

Note that a second order differentiator is used instead of a first order one, the variable z_2 is not used, the main reason for this selection is to reduce the presence of numerical chattering on the estimated variables.

OBE design

Taking into account the Subsystem (1) and using the jacket temperature equation T_j as coupling equation to calculate Λ , an OBE based on the generalized super-twisting algorithm [7] is proposed as follows:

HOSMO design

Once the estimation of Λ is obtained, a HOSMO of the form presented in [22] is designed based on Subsystem (2). This observer provides an estimation of the state variable C_A and the parameter ΔH .

For this case, the parameter ΔH is considered as an unknown input. It is observed that the output T has a relative degree equal to one with respect with the unknown input ΔH , therefore ΔH can be written in terms of T and its time derivative \dot{T} as follows from (2).

$$\Delta H = - \left(\frac{k_0 \hat{C}_A}{\rho C_p} e^{-\frac{E}{RT}} \right)^{-1} \left(\dot{T} - \frac{F}{V} (T_{in} - T) - \frac{\Lambda}{\rho C_p V} (T_j - T) \right)$$

Thus, calculating the time derivatives of the output T using the robust exact sliding mode differentiator (3), the HOSMO structure is

$$\begin{aligned} \dot{\hat{C}}_A &= \frac{F}{V} (C_{in} - \hat{C}_A) - k_0 \hat{C}_A e^{-\frac{E}{Rz_0}} \\ \dot{\Delta H} &= -\frac{1}{\tau} \left[\left(\frac{k_0 \hat{C}_A}{\rho C_p} e^{-\frac{E}{Rz_0}} \right)^{-1} \left(z_1 - \frac{F}{V} (T_{in} - z_0) - \frac{\hat{\Lambda}}{\rho C_p V} (\hat{T}_j - z_0) \right) + \hat{\Delta H} \right] \end{aligned} \quad (5)$$

where $\hat{\Delta H}$ is the estimation of ΔH .

Complete estimation scheme

From (4)-(3), the complete estimation scheme is given by the following system:

$$\begin{cases} \dot{z}_0 = -\lambda_0 L^{\frac{1}{3}} [z_0 - T]^{\frac{2}{3}} + z_1 \\ \dot{z}_1 = -\lambda_1 L^{\frac{1}{2}} [z_1 - z_0]^{\frac{1}{2}} + z_2 \\ \dot{z}_2 = -\lambda_2 L [z_2 - z_1]^0 \end{cases} \quad (6)$$

$$\begin{cases} \dot{\hat{T}}_j = \frac{F_{jf}}{V_j} (T_{jf} - \hat{T}_j) - \frac{\hat{\Lambda}}{\rho_j C_{pj} V_j} (T_j - z_0) + k_1 \phi_1(\tilde{T}_j) \\ \dot{\hat{\Lambda}} = k_2 \frac{\rho_j C_{pj} V_j}{\delta_j} \phi_2(\tilde{T}_j) \end{cases} \quad (7)$$

$$\begin{cases} \dot{\hat{C}}_A = \frac{F}{V} (C_{in} - \hat{C}_A) - k_0 \hat{C}_A e^{-\frac{E}{Rz_0}} \\ \dot{\Delta H} = -\frac{1}{\tau} \left[\left(\frac{k_0 \hat{C}_A}{\rho C_p} e^{-\frac{E}{Rz_0}} \right)^{-1} \left(z_1 - \frac{F}{V} (T_{in} - z_0) - \frac{\hat{\Lambda}}{\rho C_p V} (\hat{T}_j - z_0) \right) + \hat{\Delta H} \right] \end{cases} \quad (8)$$

which is a coupled system formed of seven differential equations.

Convergence analysis

The convergence analysis for the system (6)-(8) is performed by analyzing the stability of the error system.

$$\begin{cases} \dot{\tilde{z}}_0 = -\lambda_0 L^{\frac{1}{3}} [\tilde{z}_0]^{\frac{2}{3}} + \tilde{z}_1 \\ \dot{\tilde{z}}_1 = -\lambda_1 L^{\frac{1}{2}} [z_1 - z_0]^{\frac{1}{2}} + \tilde{z}_2 \\ \dot{\tilde{z}}_2 = -\lambda_2 L [z_2 - z_1]^0 - T^{(3)} \end{cases} \quad (9)$$

$$\begin{cases} \dot{\tilde{T}}_j = -\frac{F_{jf}}{V_j} \tilde{T}_j + \frac{1}{\rho_j C_{pj} V_j} \left(-\tilde{\Lambda} T_j + \Lambda T - \hat{\Lambda} z_0 \right) - k_1 \phi_1(\tilde{T}_j) \\ \dot{\tilde{\Lambda}} = -k_2 \frac{\rho_j C_{pj} V_j}{\delta_j} \phi_2(\tilde{T}_j) + \dot{\Lambda} \end{cases} \quad (10)$$

$$\begin{cases} \dot{\tilde{C}}_A = -\frac{F}{V} \tilde{C}_A - k_0 C_A e^{\frac{E}{RT}} + k_0 \hat{C}_A e^{\frac{E}{Rz_0}} \\ \dot{\Delta \tilde{H}} = -\frac{1}{\tau} \left[\left(\frac{-k_0 \hat{C}_A}{\rho C_p} e^{\frac{E}{Rz_0}} \right)^{-1} \left(z_1 - \frac{F}{V} (T_{in} - z_0) - \frac{\hat{\Lambda}}{\rho C_p V} (\hat{T}_j - z_0) \right) + \Delta \hat{H} \right] \end{cases} \quad (11)$$

where

$$\tilde{z}_0 = z_0 - T, \tilde{z}_1 = z_1 - \hat{T}, \tilde{z}_2 = z_2 - \ddot{T}, \tilde{\Lambda} = \Lambda - \hat{\Lambda}, \tilde{C}_A = C_A - \hat{C}_A$$

$$\text{and } \tilde{T}_j = T_j - \hat{T}_j.$$

At first, for the system (9), with the third derivative $T^{(3)}$ be bounded as $|T^{(3)}| < \delta_T$ for all the time and

$L > \delta_T$, the errors \tilde{z}_0 , \tilde{z}_1 and \tilde{z}_2 converge to zero in a finite time $t_j > 0$ [24].

Now, for the system (10), let $q = \frac{1}{\rho_j C_{pj} V_j} \left[-\tilde{\Lambda} T_j + \Lambda T - \hat{\Lambda} z_0 \right]$, hence

$$\begin{aligned} \dot{q} &= \frac{1}{\rho_j C_{pj} V_j} \left[-\dot{\tilde{\Lambda}} T_j + \tilde{\Lambda} \dot{T}_j + \dot{\Lambda} T + \Lambda \dot{T} - \dot{\hat{\Lambda}} z_0 - \hat{\Lambda} \dot{z}_0 \right] \\ &= \frac{1}{\rho_j C_{pj} V_j} \left[-\left(\dot{\Lambda} - \dot{\hat{\Lambda}} \right) T_j - \tilde{\Lambda} \dot{T}_j + \dot{\Lambda} T + \Lambda \dot{T} - \dot{\hat{\Lambda}} z_0 - \hat{\Lambda} \dot{z}_0 \right] \\ &= \frac{1}{\rho_j C_{pj} V_j} \left[-\left(z_0 - T_j \right) \dot{\hat{\Lambda}} + \left(T - T_j \right) \dot{\Lambda} + \tilde{\Lambda} \dot{T}_j - \Lambda \dot{T} - \hat{\Lambda} \dot{z}_0 \right] \end{aligned}$$

Therefore, from (4) and (10), it follows:

$$\begin{aligned} \dot{\tilde{T}}_j &= -\frac{F_{jf}}{V_j} \tilde{T}_j + q - k_1 \phi_1(\tilde{T}_j) \\ \dot{q} &= k_2 \theta \phi_2(\tilde{T}_j) + \Delta \end{aligned} \quad (12)$$

where $\theta = \frac{z_0 - T}{\delta_j} > 1$ and

$\Delta = \frac{1}{\rho_j C_{pj} V_j} \left[\left(T - T_j \right) \dot{\hat{\Lambda}} + \tilde{\Lambda} \dot{T}_j - \Lambda \dot{T} - \hat{\Lambda} \dot{z}_0 \right]$ is an unknown but bounded perturbation, $|\Delta| < \delta_q$ for all

the time, with δ_q a positive constant.

If the gains k_1, k_2 are in the set the system (12) is fixed-time stable [7], that is a sliding mode appears on the manifold $(\tilde{T}_j, q) = (0, 0)$ in a fixed time $t_q > 0$.

$$K = \left\{ (k_1, k_2) \in \mathfrak{R}^2 \mid 0 < k_1 \leq \sqrt{\delta_q}, k_2 > \frac{k_1^2}{4} + \frac{4\delta_q^2}{k_1^2} \right\} \cup \left\{ (k_1, k_2) \in \mathfrak{R}^2 \mid k_1 > 2\sqrt{\delta_q}, k_2 > 2\delta_q \right\}$$

In this form, with $t_{jq} = \max(t_j, t_q)$, a sliding mode appears on the manifold $(\tilde{z}_0, \tilde{z}_1, \tilde{z}_2, T_j, q) = (0, 0, 0, 0, 0)$ in a finite time $t_{jq} > 0$.

The motion on the sliding manifold $(\tilde{z}_0, \tilde{z}_1, \tilde{z}_2, T_j, q) = (0, 0, 0, 0, 0)$ is given by

$$\dot{\tilde{C}}_A = -\frac{F}{V}\tilde{C}_A - k_0\tilde{C}_A - k_0\tilde{C}_A e^{-\frac{E}{RT}} \quad (13)$$

$$\dot{\hat{\Delta H}} = -\frac{1}{\tau} \left[\left(\frac{-k_0\hat{C}_A}{\rho C_p} e^{-\frac{E}{Rz_0}} \right)^{-1} \left(\dot{T} - \frac{F}{V}(T_{in} - T) - \frac{\Lambda}{\rho C_p V}(T_j - T) \right) + \hat{\Delta H} \right] \quad (14)$$

Taking into account that (14) is just a first order low-pass filter for the estimation of ΔH given by

$$\left(\dot{T} - \frac{F}{V}(T_{in} - T) - \frac{\Lambda}{\rho C_p V}(T_j - T) \right)$$

the stability analysis is completed considering the equation (13). Thus, let the Lyapunov function

$$\begin{aligned} \dot{V} &= \tilde{C}_A \left(-\frac{F}{V}\tilde{C}_A - k_0\tilde{C}_A e^{-\frac{E}{RT}} \right) \\ &= -\frac{F}{V}\tilde{C}_A^2 - k_0\tilde{C}_A^2 e^{-\frac{E}{RT}} \\ &< -\frac{F}{V}\tilde{C}_A^2 \end{aligned}$$

Therefore, the error \tilde{C}_A is exponentially stable to the equilibrium point zero.

Numerical Simulation Results

The numerical simulation results of the proposed estimation structure applied to a CSTR

are presented in this section. All simulations presented here were conducted using the Euler integration method, with a fundamental step size of 1×10^{-4} [min]. The CSTR parameters are shown on Table 1.

Table 1. Nominal Parameters of the CSTR.

Parameter	Values [Unit]	Parameter	Values [Unit]
F	0.1605 [m ³ min ⁻¹]	C_p	3571.3 [Jkg ⁻¹]
V	2.4069 [m ³]	U	2.5552×10^4 [J(minm ² K) ⁻¹]
C_{in}	2114.5 [gmolm ³]	A	8.1755 [m ²]
k_0	2.8267×10^{11} [min ⁻¹]	F_{jf}	0.3376 [m ³ min ⁻¹]
E	75361.14 [Jgmol ⁻¹]	V_j	0.24069 [m ³]
R	8.3174 [Jgmol ⁻¹ K ⁻¹]	T_{jf}	279 [K]
T_{in}	295.22 [K]	ρ_j	1000 [kgm ³]
ΔH	-9.0712×10^4 [Jgmol ⁻¹]	C_{pj}	3728.87 [Jkg ⁻¹]
ρ	1000 [kgm ⁻³]		

In order to verify the observer performance in presence of parametric variations, the changes shown in Table 2 were introduced in the simulation. It is worth to notice that these changes are unknown for the proposed estimation system.

Table 2. CSTR Parametric Variations.

Parameter	Variation Time	Variation Magnitude
ΔH	45min	10% Negative step
A	100min	50% -negative exponential

The initial conditions for the CSTR were selected to: $T_j(0)=284.084[K]$, $T(0)=314.7208[K]$ and; $C_A(0)=907.1774[gmolm^{-3}]$ and for the observer to: $\hat{T}_A(0)=286.9248[K]$, $\hat{\Lambda}(0)=1.8801 \times 10^5 [J.(minK)^{-1}]$, $\hat{C}_A(0)=997.8951[gmolm^{-3}]$, $z_0(0)=314.7208[K]$, $z_1(0)=0[Kmin^{-1}]$, $z_2(0)=0[Kmin^2]$ and $\hat{\Delta H}(0)=-9 \times 10^4 [Jgmol^{-1}]$. For the OBE the selected

parameter values were adjusted to: $k_j=3.4026$, $k_2=5.9821 \times 10^4$ and $\mu=1$. For HOSMO, the parameters $\lambda_0=2$, $\lambda_1=1.5$, $\lambda_2=1.1$ and $L=10$ were used. Finally, the filter constant was adjusted to: $\tau = \frac{1}{2}$.

This section is divided into two parts. In the first part, there is assumed that the temperature measurements are noiseless; in the second part instead, there is included a normally distributed random signal as measurement noise in the temperature.

Noiseless measurements

In this subsection, there is assumed no noise in the measurements.

Figures 2 to 4 show the concentration of reactive inside the reactor C_A , the heat of reaction ΔH and the global coefficient of heat transfer A , respectively, for noiseless measurements.

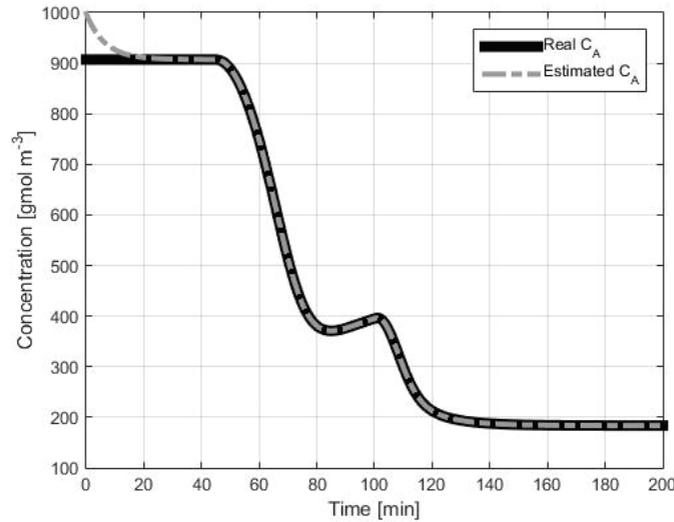


Figure 2. Concentration of the reactive inside the reactor CA (real and estimated).

The estimated concentration converges to the actual concentration, with a settling time of approximately 20min (see Figure 2). Then for changes in both parameters it can be noted that the estimate of the concentration remains correct ($t=45min$ y $t=100min$), which shows that the scheme of Figure 1 is robust with the parametric

changes. That is, for parametric changes the observer responds correctly, estimating the variable of interest without steady-state error. Likewise, Figures 3 and 4 show the correct estimation of the parameters for changes in the initial condition and changes in the process.

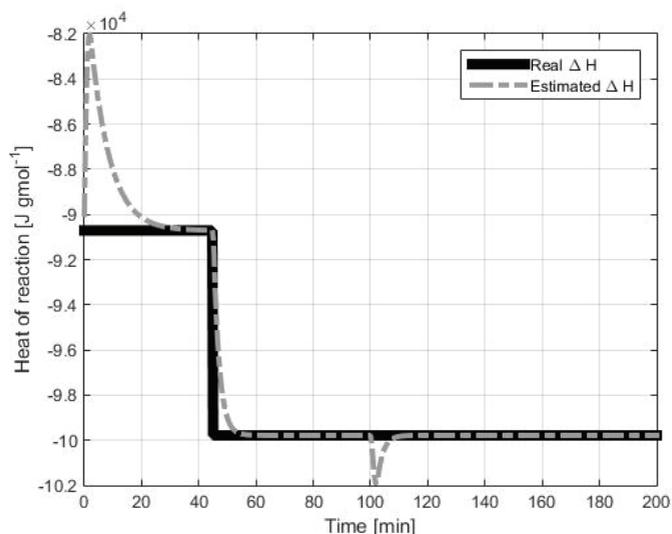


Figure 3. Heat of reaction ΔH (real and estimated).

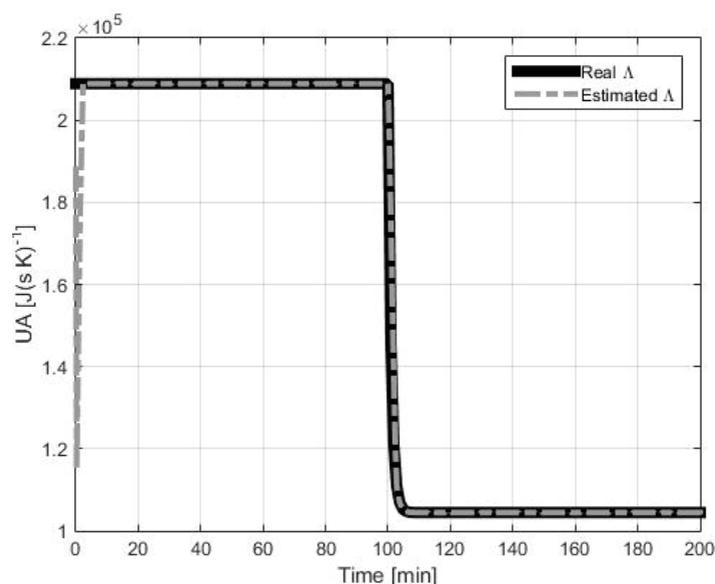


Figure 4. Global co efficient of heat transfer A (real and estimated).

Noisy measurements

In this subsection, there is assumed that the temperature measurements were corrupted by a normally distributed random signal with zero mean and 0.1111 variance; this assumed variance responds to the fact that many temperature sensors report an accuracy of $\pm 1[K]$.

Figures 5 to 8 show the temperature inside the jacket T_j , the concentration of reactive inside the reactor C_A , the heat of reaction ΔH and the global coefficient of heat transfer A , respectively, for noisy measurements.

Based on the presented figures, a good

performance of the proposed scheme is observed, also under noisy measurement conditions.

The measurement noise is filtered by the estimation algorithms (Figure 5), this way the estimated concentration convergence is similar to the noiseless convergence.

Also the fast and accurate convergence of the estimated parameters can be highlighted, although a little deviation is noted in the estimated heat of the reaction due to the high gain of the observer for this case. The HOSMO presents a correct estimation due to its capacity to calculate the parameters (Figures 7 and 8).

In addition, the simultaneous estimation of ΔH and λ using the OBE makes the proposed system robust against this parameter variations under noise conditions, the estimated temperature inside the jacket is much closer to its real value than its measurement (Figure 5) and the estimated concentration remains correct, without steady-state error. Similarly to the noiseless measurements

case the scheme is robust with the parametric changes.

Besides, it can be noted that the HOSMO presents reduced chattering due to the use of the second order differentiator; with this proposal, all discontinuities that produce chattering come up at the second time derivative, not at the first which is used in the estimation.

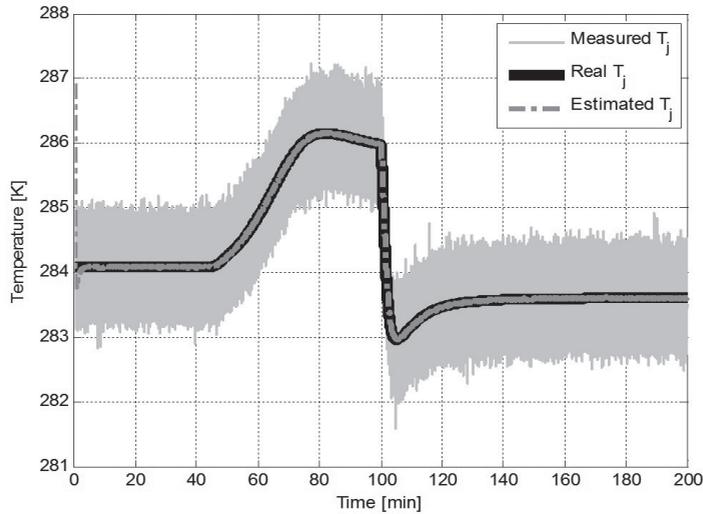


Figure 5. Temperature inside the jacket T_j (real, estimated and measured).

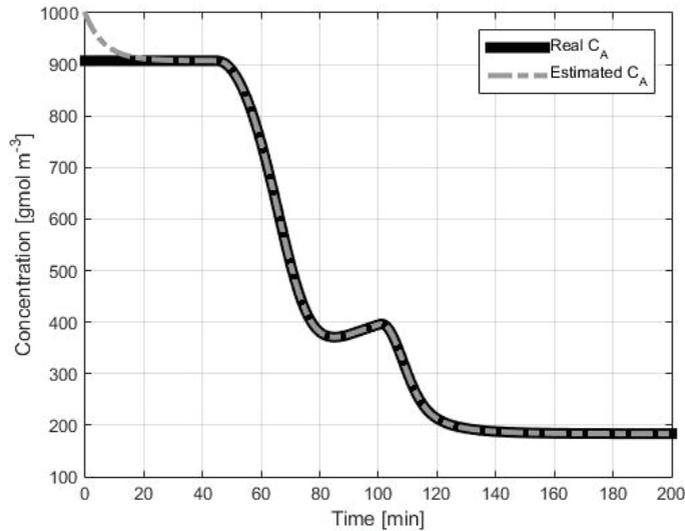


Figure 6. Concentration of the reactive inside the reactor C_A (real and estimated).

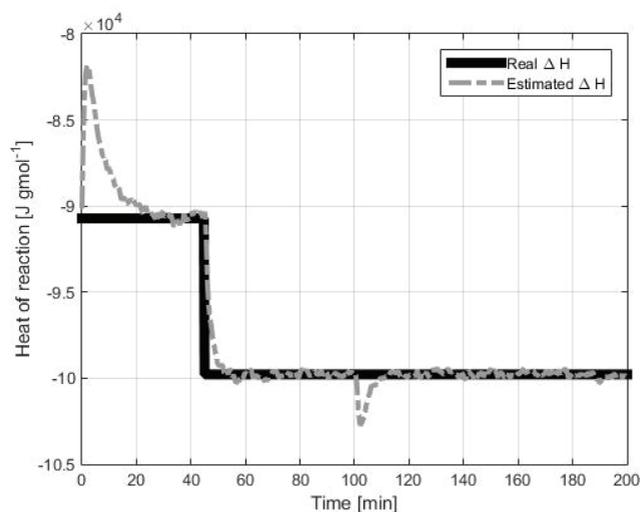


Figure 7. Heat of reaction ΔH (real and estimated).

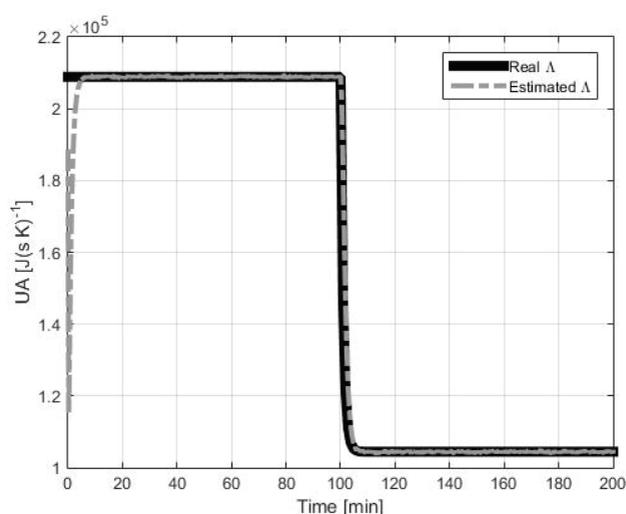


Figure 8. Global coefficient of heat transfer Δ (real and estimated).

Conclusions

An estimation structure for a CSTR based on a second order sliding mode extension of an OBE coupled to a HOSMO was proposed. This scheme, apart from allowing state estimation, allows estimating two parameters of the process, which are well known as very difficult to measure, especially for real-time applications. One of these parameters was considered as an unknown parameter in the HOSMO, and the second one is estimated by a second order sliding mode OBE. Hence, this configuration ensures a state estimation that is robust against variations of the mentioned parameters. With a suitable selection of the

observer parameters, the structure presents a fast convergence with high accuracy.

Numerical simulations show a good performance of the proposed approach, exposing some features as the accurate estimation of both parameter and state, robustness against these parameter variations, and a short-time convergence of the whole system, also under noisy-measurement conditions. Furthermore, the proposed estimation scheme presents reduced chattering presence due to the use of a second order differentiator instead of a first order one. The proposed approach can be extended to more complex processes with similar structures. For these cases, the computational burden can increase.

Acknowledgements

Juan Diego Sánchez acknowledges to CONACyT, México for the PhD scholarship number 240280 and the support of Proyecto de Movilidad Internacional de la Diáspora Científica de Alto Reconocimiento, año 2012 Colciencias - Banco Mundial to his internship in Colombia.

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