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Quartz grinding specific rate of breakage (Sj) classification by discriminant analysis

Clasificación de la velocidad específica de fractura (Sj) mediante análisis discriminante para la molienda de cuarzo

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Abstract

Specific rate of breakage (Sj) is an important parameter for grinding kinetics behavior due to it is reverse related with the process energy consumption. Size grinding media, viscosity medium, and fine particle formation are some of modifiable variable for to reduce the energy in the grinding process. Nowadays, there is no model that explains the relationship among Sj and parameters described above. A classification model based on linear discriminant analysis for quartz wet grinding was proposed to identify conditions with the high Sj. Three grinding kinetic behavior groups have been found through cluster analysis and two discriminant functions that explicate difference among groups. The first function was the most powerful differentiating dimension with 89.01% of prediction percentage, and the second one represented an additional significant dimension with 10.99% of prediction.

Keywords: ball milling; discriminant analysis; grinding; quartz; specific rate of breakage.

Resumen

La velocidad específica de fractura (Sj) es un parámetro determinante para el comportamiento cinético de la molienda debido a la relación inversa que tiene con el consumo energético del proceso. Tamaño de bolas, viscosidad del medio y formación de partículas finas son algunas variables que se pueden modificar para reducir el consumo energético en los procesos de molienda. No obstante, no existe un modelo que explique la relación entre la Sj y los parámetros descritos anteriormente. Se propone un modelo de clasificación basado en el análisis discriminante para identificar las condiciones que permitan obtener las mayores Sj en una molienda húmeda de cuarzo. Se obtuvieron tres grupos del comportamiento cinético mediante el análisis de clúster y dos funciones discriminantes que explican la diferencia entre los grupos. La primera función discriminante fue la más poderosa con 89.01 % de predicción y la asegunda función representa la dimensión adicional con un 10.99% de predicción.

Palabras clave: análisis discriminante; cuarzo; molienda; molino de bolas; velocidad específica de fractura.

1. Introduction

Grinding is often the most important stage in a minerals processing plant, due to the fact that this process generates, through fracture events, minerals liberation than can later be separated by concentration processes, which is a high energy consumption process, accounting between 30-50% of total mineral processing plant [1],

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[2]. A fine size obtained by ball mill grinding usually is carried out by wet or dry processes; the difference lies in terms of energy consumption, inasmuch as dry grinding is around 15-50% higher than wet grinding [3]–[5]. Wet grinding modifies breakage behavior and energy condition of the milling process [6], [7], presenting it differently to dry grinding. When a grinding process is carried out, the small particles size formation increases the number of particles for a constant volume inside ball mill, the formation of fine particles will transform the flow behavior from Newtonian fluid to shear thinning or to shear thickening fluid. Wet grinding has been implemented using quartz ore as a reference because it covers some of the most commercial value minerals such as gold and silver [8], and using a Newtonian polyacrylamide (PAM) solution as a suspending medium to prepare the final slurry . The use of PAM solutions are extensively used as chemical additives or processing aids in the manufacturing of paper [9], [10]. The first order law for batch model describes the grinding kinetic as shown in equation (1), [11], [12]. Nevertheless, rheological behavior can change grinding scenery and move from normal to abnormal region altering specific rate of breakage and energy efficiency as well.

$$\frac{dw_i(t)}{dt} = \sum_{j=1}^{i-1} b_{ij} S_j w_i(t) - S_j w_i(t) \tag{1}$$

The analysis of equation (1) for wet grinding does not include some parameters, resulting in a bivariable correlation [8], [2].

A linear discriminant analysis (LDA) is proposed in order to find relationships that involve rheological variables in the breakage kinetic behavior, to achieve process optimization and proper models to describe specific rate of breakage, which is the most important variable in breakage kinetic. To achieve accuracy it, was necessary to define principal component analysis (PCA), -a mathematical tool employed as an exploratory data technique to obtain an overview of the variable-. PCA transforms the original correlated data into uncorrelated new variables to reduce the dimension to interpret data while retaining as much information present in the original dataset as possible [13]. A cluster analysis (CA) was used to search patterns in the dataset by grouping the observations into clusters. This technique allows finding a natural structure among objects. By grouping individuals, the goal of each cluster obtained is to maximize homogeneity within the groups and heterogeneity among groups based on the response variable distance [13], [14].

LDA aims at seeking discriminant projection axes that separate patterns with different class labels, in this case, discriminant analysis is used to identify some individuals that exhibit different behavior in terms of breakage kinetics or normal fracture behavior. The discriminant function uses two criteria to create a new axis [13]:

1. Maximize the distance between means of the classes.
2. Minimize the variation within each class

The first function maximizes the differences between the values of the dependent variable. The other functions maximize the differences between values of the dependent variable, controlling for the first factor [15].

2. Methods and materials

2.1. Experimental test

Wet grinding of quartz (SiO_2) ore was carried out in a laboratory steel ball mill, Figure 1. The apparent viscosity of the slurry was determined using a Brookfield DV-II+Pro viscosimeter with a RV-1 HA/HB-1 spindle which shear rate is 66 s^{-1}, the size of product passing 80% (P_{80}) of accumulated mass using the Schuhmann adjustment [16], the Grindability Index (GI) using the Napier Munn criteria [6], Bond Work Index (BWI) with Bond mill [17], and the specific rate of breakage (S_j) through a monozise test [2]. In total 36 kinetic experiments were developed under different operating conditions. See Table 1.

Figure 1.Wet grinding process in a laboratory ball mill.
Source: Mill room SIU - UdeA

2.2. Materials

A typical quartz from Antioquia, Colombia was used for this research; its chemical composition was determined by X ray fluorescence (XRF) and presented in Table 2. Three different sizes were employed, 53, 45, and 38 μm, corresponding to mesh 270, 325, and 400 [18], respectively. Polyacrylamide (PAM) solutions of, 0.00, 0.02, 0.035, and 0.05 % percentages w/w, prepared with drinking water were used as dispersant agent. Finally, monodispersed suspensions of quartz were prepared with PAM solutions at 60% percentage w/v of solids for each size.

Table 1. Wet grinding conditions

Materials	**Quartz**
Solids concentration, ϕ (%w/v)	60
Mill length, L (m)	0.18
Mill diameter, D (m)	0.16
Ratio L/D	1.16
Fraction of critical speed, ϕ_C	0.75
Ball filling fraction, J	0.3
Hole fraction, U	1.0
Bed normal porosity	0.4
Slurry viscosity (cp)	1, 4,6,8
Feed size (µm)	53, 45, 38
Liquid	Polyacrylamide solution

Source: Own source.

Table 2. Chemical composition of Quartz

Oxides	**SiO_2**	**Al_2O_3**	**Fe_2O_3**	**TiO_2**
Percentage (%)	94-96	2.0-4.5	0.2	0.2
Oxides	**CaO**	**MgO**	**Na_2O**	**K_2O**
Percentage (%)	0.1	0.1	0.1	0.1

Source: Spectroscopy laboratory-UdeA.

2.3. Statistical Analysis

A statistical analysis was performed with an open statistical software, R studio 3.6.1. A principal component analysis (PCA) was used to reduce dimensions and to compare different variables involved in the study and to look for the correlation among them. To find grouping observations, the cluster analysis (CA) was proposed and finally a linear discriminant analysis (LDA) was employed as supervised method to represent a sample from the population and obtain discriminant functions for define the best kinetic setting of breakage as show in Figure 2.

3. Results and discussions

In Table 3, the experimental test summary of some of the kinetic tests, which allowed determining five fundamental different variables after 216 grinding tests and 36 kinetic tests for the statistical analysis, is shown.

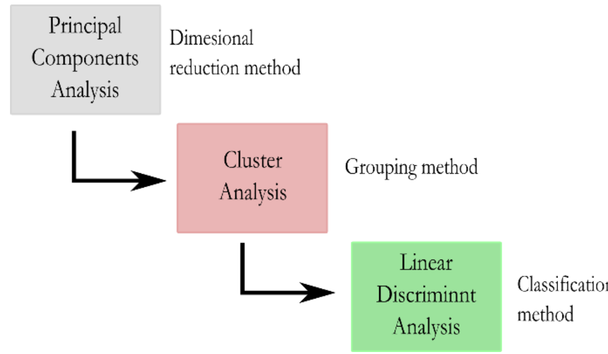

Figure 2.Statistical methodology to determine LDA. Source: Own source.

Table 3.Summary of different kinetic tests

Kinetic tests	**1**	**25**	**36**
P_{80}(µm)	44.41	41.50	36.14
S_j(min^{-1})	0.205	0.199	0.053
Ap.viscosity(cp)	25	60.35	61.9
Bond Work Index (kwh/ tonc)	15.17	7.49	6.73
Grinding Index	86.79	60.01	35.35

Source: Own source

3.1. Principal components analysis (PCA)

The principal component analysis (PCA) results of 5 variables in a wet grinding for 36 tests are shown in Table 4. It is necessary to select the number of components so the accumulated variance is at least 80 % or the eigenvalues are greater than one [19]. For this analysis, two principal components describe the behavior of five initial variables as observed in equations (2) and (3).

The first principal component (PC1) is highly correlated with breakage function (Sj), Bond work index, and Grinding Index loading, and explains 57,39% of the total variance, which indicates that PC1 can represent the effect of overall breakage of particles across the mill in the grinding process. PC2 accounts for 26.95% of the total variance and it evidences an inverse relation between fine particle formation (P_{80}) and relative viscosity of grinding scenery. It can be mentioned that the second principal component include the rheological information due to the fact that, the hold up of slurry remains in steady state for a batch grinding, but the particle size distribution changes during the residential time inside the mill. Therefore, the small particle formation may cause a hydrodynamic effect, which increases the viscosity, improves the particle-particle interactions, resulting in a high flow resistance and a great apparent viscosity [20].

Table 4. Rotated factor loadings of principal components

Variables	Dim. 1	Dim. 2	Dim. 3	Dim. 4	Dim. 5
P_{80}	-0.37	0.55	-0.53	-0.48	0.21
S_j	-0.52	-0.1	-0.42	0.74	-0.07
Viscosity	-0.01	-0.8	-0.46	-0.37	0.06
Bond Work Index	-0.56	-0.08	0.3	-0.3	-0.71
Grinding index	-0.53	-0.19	0.49	-0.06	0.66
Eigenvalues	2.90	1.95	0.46	0.24	0.08
Cumulative variance (%)	57.4	84.3	93.4	98.3	100

Source: Own source

$$PC1=0.3747\ P_{80}-0.5197\ Sj-0.0120\ \mu\ -0.5559\ BWI\ -0.5293GI \quad (2)$$

$$PC2=0.5502\ P_{80}\ -0.0973\ Sj-0.8029\ \mu\ -0.0798\ BWI\ -0.1918GI \quad (3)$$

3.2. Hierarchical cluster analysis (HCA)

Cluster analysis goal is to group individuals. Each cluster obtained assure a maximal homogeneity withing groups and maximal heterogeneity among groups based on the distance [13]. To define the number of clusters of variables due to similarity, a hierarchical cluster analysis was performed based on principal components analysis (PCA). In this case, the grouping was formed with the results obtained in 36 kinetic tests. As shown in Figure 3, the phenon line -imaginary horizontal line-, was defined in 20, forming 3 groups of different kinetic behaviors.

3.3. Linear Discriminant Analysis (LDA)

The linear discriminant analysis was used to identify kinetic behaviors of quartz slurry, to improve wet grinding process, and to seek a discriminant function to separate the groups, which characterized the milling scenery and to find hidden patterns among samples on dataset (Figure 4).

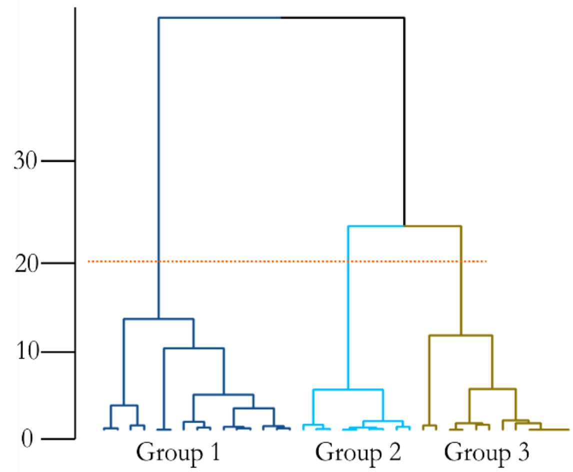

Figure 3. Cluster analysis. Source: Own source.

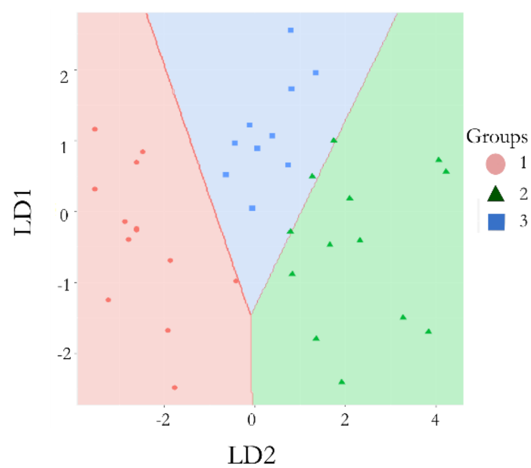

Figure 4. The statistical analysis found the existence of three groups, and clearly shows how each kinetic test has a different behavior.

As shown Figure 5, the group one clustered the maximum grindability index, the lowest apparent viscosity, the biggest P_{80}, highest specific rate of breakage, and the highest Bond work index, considering a normal breakage region. This group contained the highest feed size of the population and according with the failure theory, it is easier to break greater size than lower size particles. Group two shows a transitory behavior containing the minimum grindability index, but in this case the apparent viscosity starts to increase, whilst P_{80} decreases and the specific rate of breakage and Bond work index present the lowest value for this group, situation not common in grinding process because Bond work index would be expected to increase as long as there exists a low capture probability. It can be said that the particle size and PAM concentration do not present a good relationship, favoring the runoff of particle effect inside the mill, consequently, the kinetic scenery would have an abnormal behavior and the breakage rate is not described by first order equation. Finally, group three collects the maximum grindability index, the highest apparent viscosity, the lowest P_{80}, and a high specific rate of breakage with middle size particles. It is observed that the reduction of the particle size of solids promotes an increase in the apparent viscosity of the slurry [20]; however, the kinetic parameters are not affected by the rheological behavior, ensuring for this group optimal breakage in wet grinding conditions without a loss in the specific rate of breakage for this group, thus staying in the normal region.

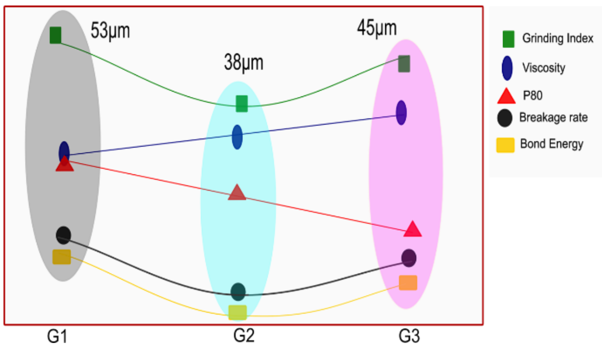

Figure 5. Groups obtained by LDA. Source: Own source.

Equations (4) and (5) show the coefficients of the linear discriminant analysis which allows differencing among groups it also show the proportion of trace or separation percentage achieved by each discriminant axes. LD1 is strongest with 89.01% of the separation, in this case, the second discriminant LD2, supplements the separation criterion with 10.99%.

$$LD1=1.301PC1-0.382PC2 \qquad \text{Trace:}89.01\% \qquad (4)$$

$$LD2=-0.1404PC1-1.017PC2 \qquad \text{Trace:}10.99\% \qquad (5)$$

4. Conclusions

- Discriminant analysis allows classifying a wet grinding process in three groups according to different breakage kinetic behaviors, namely, group one and group three have a normal breakage region, instead group two exhibit an abnormal breakage region.
- Linear discriminant analysis identified hidden pattern in a wet grinding process, which involves rheological variables and the overall breakage of particles parameter inside the mill.
- Although group three evidenced a viscosity increased tendency, the specific breakage rate, Bond Work Index, and Grinding Index have a similar order to group one.
- The group three viscosity increases in comparison with group one. But this difference neither affects the specific rate of breakage under different grinding conditions nor the normal region classification.

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