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Sensitivity analysis of a member under compression via Monte Carlo method

Análisis de sensibilidad de un miembro a compresión vía el método de Monte Carlo

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Abstract

The present work studies the application of a probabilistic methodology in the sensitivity analysis of a steel column to identify the dominant parameters in its load capacity. Monte Carlo type simulations, in combination with the finite element method, was carried out to achieve the proposed objective. The geometric nonlinearity in the model was considered in order to reflect large deflections and initial geometric imperfections. The results show that the sensitivity of the column to a specific input parameter depends on the slenderness ratio and, hence, the column will be more sensitive to one parameter or another depending on that relationship.

Keywords: buckling load; finite element analysis; stochastic modelling; modal analysis.

Resumen

En este trabajo se presenta un estudio probabilístico para el análisis de sensibilidad de una columna de acero, con el fin de identificar los parámetros que más afectan su capacidad de carga. Se llevaron a cabo simulaciones tipo Monte Carlo, en combinación con simulaciones numéricas, mediante el uso del método de elementos finitos. En el modelo numérico, se consideró la no linealidad geométrica, con el objeto de considerar grandes desplazamientos e imperfecciones geométricas iniciales. Los resultados muestran que la sensibilidad de la columna a una variable de entrada específica depende de la relación de esbeltez, y, por lo tanto, la columna será más sensible a una variable u otra en función de esa relación.

Palabras clave: carga de pandeo; análisis por elementos finitos; modelado estocástico; análisis modal.

1. Introduction

The stochastic/probabilistic nature of the loading conditions, geometry and material properties in engineering design problems has led to the development, over the years, of probabilistic methods that consider the effect of the intrinsic uncertainties in the variables on the behaviour of structural elements. Presently, the technical literature offers methods in which the probabilistic approach is used to address the lack of homogeneity of

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parameters in structural analysis problems [1, 2, 3]. In fact, with the availability of high-speed computers, statistical simulations based on Monte Carlo methods can be performed to determine the probabilistic characteristics of the mechanical behaviour of a certain structure [4]. Moreover, with the progress of software for finite element modelling and analysis, we have tools that allow analysing complex structures [5, 6], and at the same time, permit to carry out probabilistic studies that allow, for example, to investigate the response of a structural model to the uncertainty of a particular input variable. In this regard, the present work has the purpose to illustrate the application of a probabilistic approach to performing a sensitivity analysis of a steel column with simply supported ends in order to identify the dominant parameters that affect its ultimate strength.

2. Finite element model

A finite element model of the column was developed using ANSYS [7]. Beam type elements BEAM23 of the ANSYS library [7] were used for the discretization of the domain, and only half of the column was modelled due to symmetry in the geometry and in the boundary conditions (Figure 1). Moreover, in order to properly characterise the modal shapes of the buckling problem, a total of 10 elements were used. Additionally, the material has an elastoplastic behaviour, and the nonlinear response of the column was traced employing the modified Riks method [8].

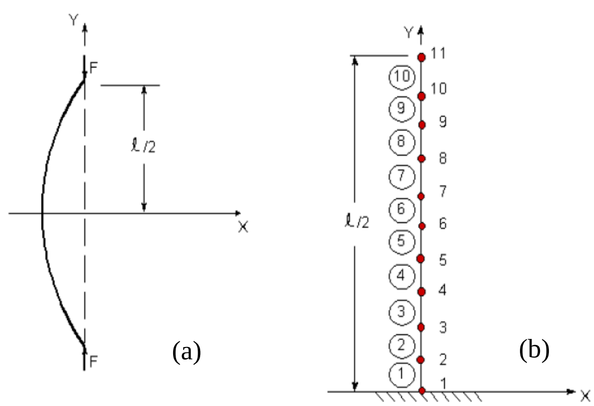


Figure 1. (a) Scheme of the studied column, (b) finite element model.

Table 1 compares the ultimate load obtained through nonlinear finite element analysis (P_{EF}) with the critical load (P_{CR}) calculated from the Euler formula for columns with articulated ends and different effective lengths (L_e). Table 2 shows the mean values of the input parameters (diameter, modulus of elasticity, yield strength, etc.) used in these calculations. Note that the difference between the values obtained from the finite element analyses with those given by Euler's formula is due to the fact that Euler's formula is based on the theory of ideal columns, where the geometry is perfect, only small deflections are considered, and the material obeys Hooke's law. However, the problem analysed herein concerns a compression member initially deflected by a small amount, with an elastoplastic behaviour and with the possibility of undergoing large deflections.

Table 1. Comparison between numerical and theoretical (Euler) results.

L_e [m]	P_{FE} [kN]	P_{CR} [kN]	Δ [%]
2	700	818	14.42
2.5	477	524	8.96
3	339	364	6.86
4	195	204	4.41

3. Modeling of geometric imperfections

Two basic considerations were taken into account to model geometric imperfections: First, the patterns of initial imperfection were characterised as a sum of the first three modes of instability, this means that the forms of imperfection simply correspond to a linear combination of those buckling modes. Another consideration is the fact that the maximum amplitude of imperfection at any point in the model is restricted to a value specified by the design codes. Herein, a mean value of imperfection amplitude of $L/1500$ was used according to reference [9].

The above considerations allow to introduce imperfections in the finite element model according to the following equation:

$$W_{N_n x N_{gl}} = \sum_{i=1}^{n} (V_i)_{N_n x N_{gl}} w_i \quad (1)$$

where W is the matrix of imperfections of the structure, N_n is the number of nodes to move, N_{gl} is the number of imperfect degrees of freedom, which for the member under study is limited to a displacement in the direction perpendicular to its axis in the plane of buckling and a rotation about the axis perpendicular to such buckling plane. V_i is the matrix of modal forms and w_i corresponds to the imperfection amplitudes allowed.

4. Eigenmodes of instability

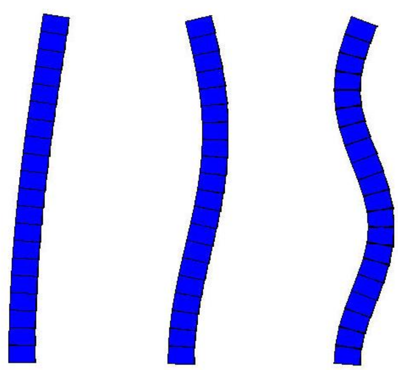
In order to obtain the modal shapes of buckling necessary to model the imperfections, it is necessary to determine the eigenmodes of instability of the column. This is performed in ANSYS [7] applying a linear perturbation procedure.

First, the stiffness matrix corresponding to the load-free state of the structure is stored to then apply a small disturbance or load. Then, the initial stiffness matrix due to the applied perturbation is processed with ANSYS, and a calculation of eigenvalues is performed to determine a multiplier of the load that makes the structure unstable. Mathematically, the above can be written as follows: If the elastic stiffness matrix is K_0 and the initial stiffness matrix due to a small perturbation Q is K_Δ, then the multipliers of Q or eigenvalues λ and the eigenmodes or modes of instability V_i must satisfy the following equation:

$$[K_0 + \lambda K_\Delta]V_i = 0 \tag{2}$$

From (2), critical loads are obtained by multiplying Q by λ. Obviously, for a practical problem, we are only interested in the first eigenvalue.

Critical loads and buckling modes can be obtained simultaneously in ANSYS since this program creates a small set of base vectors that define a subspace, that in turn is transformed by successive iterations into a space that contains the eigenmodes of the entire system. Figure 2 shows the modal shapes corresponding to the first three buckling modes with their respective critical loads.



(*a*) (*b*) (*c*)

Figure 2. (*a*) Mode 1, P_{CR1} = 791kN, (*b*) Mode 2, P_{CR2} = 7117kN, (*c*) Mode 3, P_{CR3} = 19794kN

5. Probabilistic model

The random nature of the geometrical properties and the material of the compression member studied is considered when regarding these properties as probabilistic variables with known statistical distributions. Indeed, the variability of the yield strength and modulus of elasticity of the hot-rolled AISI 1035 steel was obtained from reference [10]. Moreover, the variability in the cross-section of the column is expressed through manufacturing tolerances of the diameter of the cross-section as indicated in the AISI manual [11]. Additionally, the initial curvature of the structure is modelled according to Eq. (1) with a w_i amplitude given by a Gaussian distribution. Table 2 summarises the statistical characteristics of each random input variable.
In this table, the term GAUSS denotes a normal distribution with a mean value and a standard deviation as statistical parameters, the term WEIB indicates a Weibull distribution of three parameters, and UNIF represents a uniform distribution characterised by its lower and upper bounds.

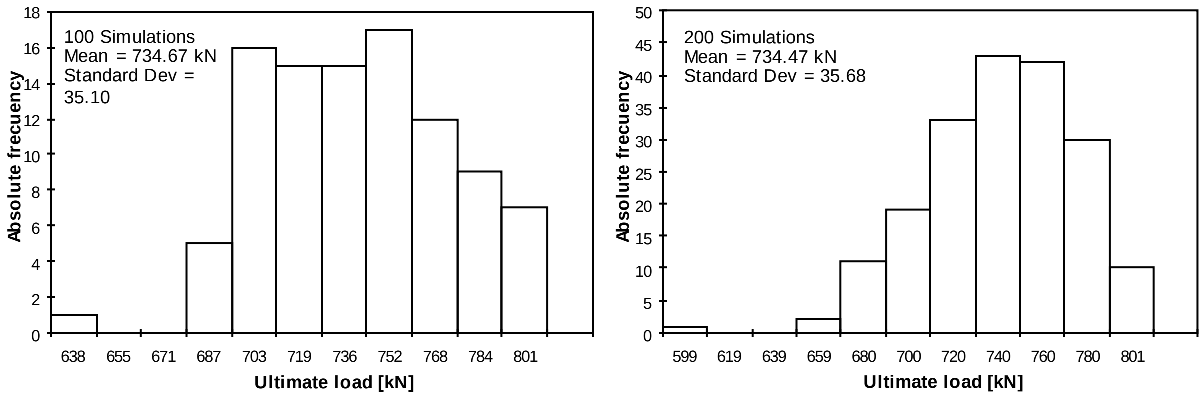
Table 2. Statistical characteristics of the input variables.

Input variable	Statistical Distribution
Diameter of the cross section [mm]	UNIF(75.94, 76.45)
Elasticity modulus [GPa]	GAUSS(206.8, 10.34)
Yield stress [MPa]	WEIB(2.88, 350, 272)
Amplitude of imperfection [m]	GAUSS(*L*/1500, *L*/4800)

6. Sensitivity analysis

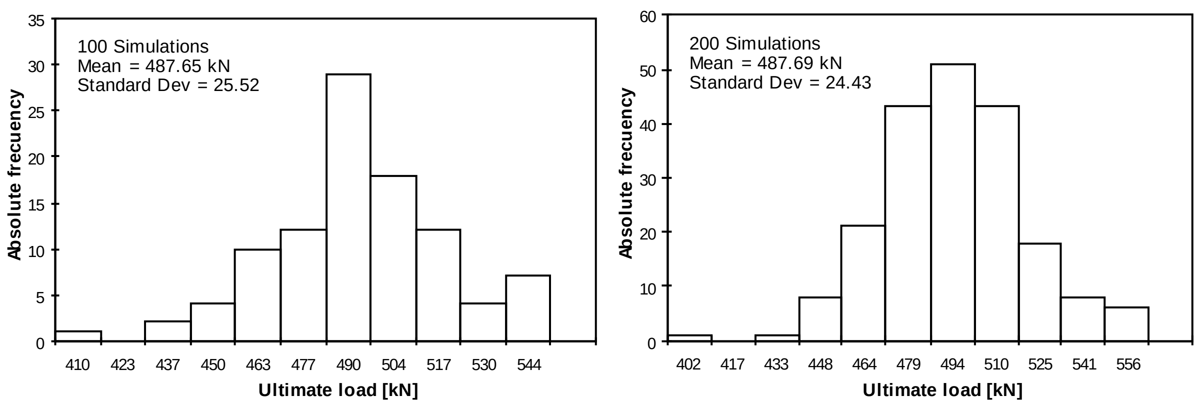
With the definition of the probabilistic model, a series of nonlinear finite element analyses were carried out in conjunction with Monte Carlo simulations to obtain the statistical data corresponding to the load capacity of the column and, then, evaluate its sensitivity to the input parameters (Table 2). Monte Carlo approach has been proven superior to other approaches for large variations of the stochastic parameters. The number of simulations needed within a Monte Carlo analysis to obtain a stable output of the results depends to a large extent on the variability of the input data. [12, 13].

Figures 3, 4, 5 and 6 show the histograms of the ultimate load for 100 and 200 simulations, respectively. With the data obtained through these simulations, we generated the sensitivity graphs that are shown in Figure 7. Notice that the data of the ultimate load of the compression member studied represents a probability density function that can be used to, subsequently, determine its structural reliability. However, the present work focuses only on the determination of sensitivities to then evaluate mechanical behaviour.



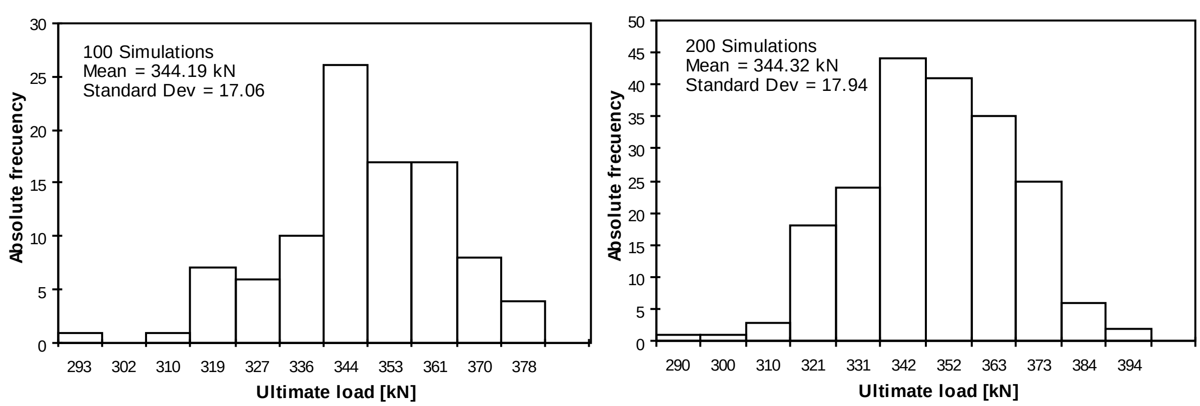
(a) (b)

Figure 3. Histogram for the ultimate load. $L_e = 2$m. (*a*) 100 simulations, (*b*) 200 simulations.



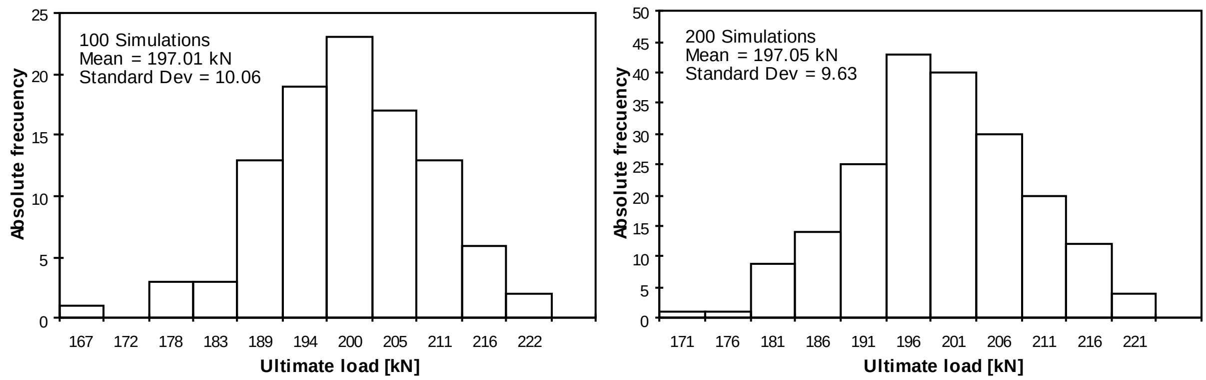
(a) (b)

Figure 4. Histogram for the ultimate load. $L_e = 2.5$m. (*a*) 100 simulations, (*b*) 200 simulations.



(a) (b)

Figura 5. Histogram for the ultimate load. $L_e = 3$m. (*a*) 100 simulations, (*b*) 200 simulations.



(a) (b)

Figura 6. Histogram for the ultimate load. L_e = 4m. (*a*) 100 simulations, (*b*) 200 simulations.

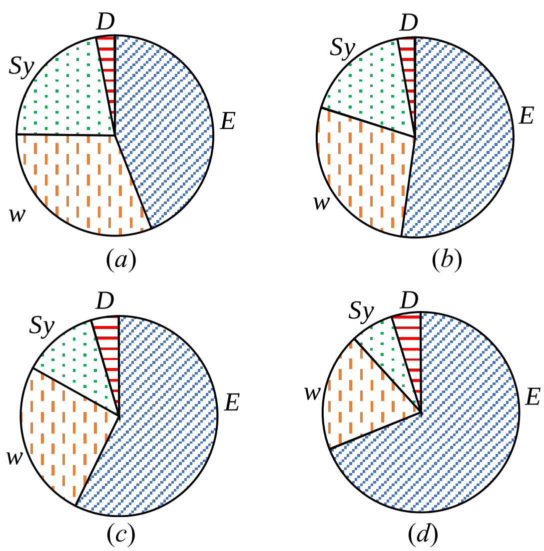


Figure 7. Sensitivity of the ultimate load for the column. (*a*) L_e = 2m, (*b*) L_e = 2.5m, (*c*) L_e = 3m, (*d*) L_e = 4m.

7. Discussion

From Figure 7, it is clearly seen that the dominant parameter in the load capacity of the compression member studied corresponds to the modulus of elasticity *E* of the material. This result was expected since it coincides with the behaviour described by column theory. It is also important to note that the effect of such parameter increases as the slenderness or length increases. However, the opposite occurs with the effect of yield strength *Sy*. This is due to the fact that, usually, very slender members will fail due to elastic instability, in contrast to more robust members where buckling occurs inelastic. Thus, the maximum load that an inelastic column can support is usually less than the Euler load for that same column (See Table 1). Furthermore, Figure 7 also indicates that the effect of the initial curvature or geometric imperfection of the compression member is of great importance. In fact, as it is known, such imperfections produce deflections from the beginning of the load and can drastically reduce the ultimate load of the column. Additionally, Figure 7 shows that the compression member is not as sensitive to a change in the tolerances of the diameter *D* of the cross-section, as long as such tolerances are within permissible limits as established in the AISI manual [6].

8. Conclusions

In this work, a rational treatment has been given to the uncertainty involved in a structural stability problem. Despite the simplicity of the problem described, the results obtained show that probabilistic procedures in conjunction with the finite element method can be used when considering the uncertainty in the mechanical behaviour of structural systems. Moreover, the presented methodology serves as a basis when determining the reliability of a structure, such that, with the simulations done, a cumulative distribution function is obtained and the probability that the structure reaches a specified ultimate load can be determined.

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