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Voltage sag state estimation based on ℓ_1-norm minimization methods in radial electric power distribution systems

Estimación de estado de hundimientos de tensión basada en métodos de minimización de la norma-ℓ_1 en sistemas radiales de distribución de energía eléctrica

Jairo Blanco-Solano[1], Nelson Kagan[2], Carlos Frederico M. Almeida[3], Johann F. Petit-Suárez[4], Gabriel Ordóñez-Plata[5]

[1]Grupo de Investigación en Sistemas de Energía Eléctrica (Gisel), Escuela de Ingenierías Eléctrica, Electrónica y de Telecomunicaciones, Universidad Industrial de Santander, Colombia. Email: jairo.blanco@correo.uis.edu.co
[2]Centro de Estudos em Regulação e Qualidade de Energia (Enerq), Departamento de Engenharia de Energia e Automação Elétricas, Universidade de São Paulo, Brasil. Email: nelsonk@pea.usp.br
[3]Centro de Estudos em Regulação e Qualidade de Energia (Enerq), Departamento de Engenharia de Energia e Automação Elétricas, Universidade de São Paulo, Brasil. Email: cfmalmeida@usp.br
[4]Grupo de Investigación en Sistemas de Energía Eléctrica (Gisel), Escuela de Ingenierías Eléctrica, Electrónica y de Telecomunicaciones, Universidad Industrial de Santander, Colombia. Email: jfpetit@uis.edu.co
[5]Grupo de Investigación en Sistemas de Energía Eléctrica (Gisel), Escuela de Ingenierías Eléctrica, Electrónica y de Telecomunicaciones, Universidad Industrial de Santander, Colombia. Email: gaby@uis.edu.co

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Abstract

Voltage sags have a high impact on the proper equipment operation and the electric power end-user processes continuity. Economic losses are a growing problem for the electric utilities, regulators and electric energy final customers and therefore, the formulation of new mathematical methods for voltage sags diagnosis are needed. In this sense, the state estimation methods seek the determination of the frequency or the number of voltage sags that an end-user would experience. In this research area, optimization problems based on techniques such as singular value decomposition, voltage profile curve fitting and voltage sag source location have been formulated. The results of these approaches may be inaccurate when the pre-fault currents, non-zero fault impedances and unbalanced conditions are considered. We will evidence that the results from singular value decomposition method are inaccurate considering these real fault conditions. Also, a new mathematical formulation of the voltage sag state estimation problem based on ℓ_1-norm minimization is proposed in this work. The proposed method is applied and validated to the IEEE 33-node test distribution network. Voltage sags caused for network faults are only considered. The results validate a remarkable improvement in comparison with the singular value decomposition method and show an innovative tool for voltage sags state estimation in radial electric power distribution systems.

Keywords: power quality; voltage sags; state estimation; ℓ1-norm; distribution networks.

Resumen

Los hundimientos de tensión tienen un alto impacto sobre la correcta operación de equipos y en la continuidad de los procesos en el usuario final de energía eléctrica. Las pérdidas económicas son un problema en crecimiento para las empresas operadoras, para los mismos reguladores y por supuesto para los consumidores finales del servicio de energía eléctrica; es así como se hace necesario la formulación de nuevos métodos matemáticos para el diagnóstico de los hundimientos de tensión. En este sentido, los métodos de estimación de estado buscan determinar la frecuencia o el número de hundimientos de tensión que experimenta un usuario final. En esta área de investigación se han formulado

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problemas de optimización basados en técnicas como la descomposición en valores singulares, el ajuste de perfiles de tensión y la localización de las fuentes generadoras de los hundimientos de tensión. Los resultados obtenidos usando estas técnicas son imprecisas cuando se consideran las corrientes pre-falla, las fallas con impedancia diferente de cero y los desbalances. Es así como en este artículo se evidenciará que, al considerar estas condiciones reales de las fallas, se obtienen resultados imprecisos para el caso del método de descomposición en valores singulares. A su vez, en este trabajo se propone una nueva formulación matemática del problema de estimación de estado de hundimientos de tensión usando la minimización de la norma-ℓ_1. Esta propuesta matemática es aplicada y validada en la red de distribución de prueba de 33 nodos del IEEE. Únicamente los hundimientos de tensión causados por fallos en la red de distribución serán considerados. Los resultados obtenidos validan una notable mejora en comparación con el método de descomposición en valores singulares y resaltan una innovadora herramienta para la estimación de estado de los hundimientos de tensión en redes radiales de distribución.

Palabras clave: calidad de potencia; hundimientos de tensión; estimación de estado; norma-ℓ_1; redes eléctricas de distribución.

1. Introduction

Voltage sags are one of the most frequent power quality disturbances. The economic losses may oscillate between the US$ 5.000 to US$ 2.500.000 per each voltage sag experienced [1], [2]. These losses are evidenced in the equipment bad-operation and the processes interruption in the end-user. Voltage sags are caused often by short circuits and several studies have been focused to analyze the voltage sag phenomena taking into account this cause [3]–[5]. In turn, voltage sag monitoring programs are still very limited and only a few busbars are monitored in the current distribution networks. A partial monitoring system can provide information to the estimation methods to determine voltage sag indices at unmetered busbars. For this, further studies are required in this research area.

Voltage Sag Estimation (VSE) is defined as the task on estimating the voltage sags number at unmetered busbars by using the collected data at a limited number of meters installed in the network [5],[6]. Two approaches were found: the methods that realize stochastic prediction and the methods based on the conventional state estimation, where measurements and simulation are used simultaneously.

A probabilistic method to obtain the most probable voltage sag index at unmetered busbars given a measurements set is presented in [8]. Voltage sag measurements are assessed into of disturbance database using a Bayesian filter. Other works also use statistical modelling to analyze the fault networks [9], [10]. The results of these methods are sensitive to the high variability of fault statistical data, so the voltage sag number may be inaccurate.

VSE problem can also be formulated as an undetermined linear system based on the fault position concept [11]. Singular Value Decomposition (SVD) technique allows to estimate the number of voltage sags in unmetered busbars. SVD method is based on the least-squares and gets a solution with minimum (Euclidean) norm. Thus, this method tends to obtain a state vector with many nonzero values. On the other hand, $\ell 1$-norm minimization method solves an undetermined linear problem while the optimal vector is sparse [12]. Many applications of $\ell 1$-norm minimization in power systems are coming due to its emerging potentiality [13], [14].

This work presents a new approach to solve the voltage sag state estimation problem based on $\ell 1$-norm minimization methods. The main contribution is a new mathematical formulation of the VSE problem and the assessment of two of the most popular solvers of ℓ_1-norm minimization problem.

This research is organized as follows. Section 2 provides a description of the SVD method and the new mathematical formulation using ℓ_1-norm minimization is explained in detail. Section 3 presents the simulation results of the SVD and proposed method for the 33-node test distribution system. Finally, conclusions are presented in section 4.

2. VSE mathematical modeling

Several mathematical and stochastic approaches have been formulated to solve the voltage sag estimation problem [9], [11], [15]. Stochastic methods are subject to high inaccuracies due to the network fault uncertainties and it is not possible to assess the optimality and convergence criteria in the results obtained. Due to this and to take advantage of the emerging technologies for the power system monitoring, this work is focused on the mathematical formulation that uses the voltage measurements in electric power distribution systems.

2.1. VSE using Singular Value Decomposition

State estimation in power systems is defined as the task of estimating the voltages and currents on the electric network. The state estimation formulation is based on a measurement vector (**b**), a measurement matrix (**A**), a state vector (**x**) and a measurement error vector (**e**). Equation (1) shows the mathematical representation when the measurement error is neglected. This formulation is frequently used in the voltage sag state estimation.

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & \dots & a_{1n} \\ \vdots & \vdots & \dots & \dots & \vdots \\ \vdots & \vdots & \dots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & \dots & a_{mn} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} \qquad (1)$$

In [11], the mathematical formulation in (1) is development to voltage sag estimation. This formulation is explained below.

2.1.1. Measurement matrix (A)

$\mathbf{A}_{mt\times n}$ is a binary matrix built by a short circuit study in the *n* fault positions on the network. The short circuit study is performed using the fault mathematical formulation or a software as NEPLAN, DIGSILENT Power Factory, PSCAD, ATP-EMTP, among others. Residual voltages in *mt* measurement points are recorded and compared with *k* thresholds. Two submatrices are derived here: $\mathbf{A_m}$ relates the residual voltages from *m* busbars metered with the *n* fault positions. $\mathbf{A_{nm}}$ relates the residual voltages from *nm* unmetered busbars with the *n* fault positions. The rules applied to derive these matrices are shown in table 1.

These matrices may also be built for each fault type for a better performance of the voltage sag estimation method.

Table 1. Values of the **Am** and **Anm** matrices.

Matrix	Rule	
$a_{m_{ij}}$	**1**	If residual voltage at busbar m_i is below thr_k when a short circuit occurs in the fault position n_j
	0	Otherwise
$a_{nm_{ij}}$	**1**	If residual voltage at busbar nm_i is above thr_k when a short circuit occurs in the fault position n_j
	0	Otherwise

Source: Own elaboration.

2.1.2. Measurement vector (b)

$\mathbf{b}_{m\times 1}$ is formed by voltage sag measurements obtained from *m* real meters installed on the network. Real residual voltages recorded are used to build **b**. Each element of **b** corresponds to an integer number obtained by counting the voltage sags number.

2.1.3. Optimization problem and voltage sag estimation

Optimization problem shown in (2) is solved using $\mathbf{A_m}$ and **b**, without any constraints.

$$minimize \quad ||\mathbf{A_m} \cdot \mathbf{x} - \mathbf{b}||_2^2 \qquad (2)$$

The optimal solution $\mathbf{x}^*$ is obtained using the deterministic approach of SVD, as shown in (3)

$$\mathbf{x}^* = \mathbf{V} \cdot \mathbf{W^{-1}} \cdot \mathbf{U'} \cdot \mathbf{b} \qquad (3)$$

where **U**, **W** and **V** are the matrices obtained from the decomposition of $\mathbf{A_m}$ using SVD. If a set of thresholds thr_k, for k=1,2,...,T, is defined, then an augmented problem can be formulated as shown in (4).

$$minimize \quad \left\| \begin{bmatrix} \mathbf{A_m}^{thr_1} \\ \mathbf{A_m}^{thr_2} \\ \vdots \\ \mathbf{A_m}^{thr_T} \end{bmatrix} \cdot \mathbf{x} - \begin{bmatrix} \mathbf{b}^{thr_1} \\ \mathbf{b}^{thr_2} \\ \vdots \\ \mathbf{b}^{thr_T} \end{bmatrix} \right\|_2^2 \qquad (4)$$

Solving the equation (4), the optimal solution $\mathbf{x^*}$ is finally used in (5) to calculate the voltage sags number $\mathbf{b_{nm}}^{thr_k}$ at unmetered busbars.

$$\begin{bmatrix} \mathbf{b_{nm}}^{thr_1} \\ \mathbf{b_{nm}}^{thr_2} \\ \vdots \\ \mathbf{b_{nm}}^{thr_T} \end{bmatrix} = \begin{bmatrix} \mathbf{A_{nm}}^{thr_1} \\ \mathbf{A_{nm}}^{thr_2} \\ \vdots \\ \mathbf{A_{nm}}^{thr_T} \end{bmatrix} \cdot \mathbf{x}^* \qquad (5)$$

2.2. Proposed Method: VSE using ℓ_1-norm minimization

Now, several issues in the VSE problem may be discussed:

- The physical interpretation of **x*** allows to conclude that for a set of voltage sags registered, all fault positions (values of the state vector **x**) cannot have failed. In real power systems, several busbars and lines have a higher fault probability that others. Thus, the SVD method tends to obtain a state vector with many non-zero values and this does not meet the actual distribution of the network faults in power systems. Therefore, the state vector should be sparse with few non-zero values.
- The optimal vector **x*** is obtained only considering the **Am** matrix. Equation (1) is an under-determined linear system and we found the first error source to estimate the actual number of network faults occurred here. If x* is not a good estimation, then the results of

(5) can be more inaccurate. Thus, we raise that a sparse optimal vector x* minimizes the error on voltage sag estimation in (5), this due to the sparse nature in the measurement matrix construction.

- The square error minimization in (2) is still an optimization goal that should be used.

According to the above, a new hypothesis may be inferred, stating that a sparse vector represents better the network fault nature and thus the voltage sags estimation may be more accurate.

The mathematical formulation of the VSE, as a sparse problem, is the main contribution of this paper. This formulation allows us to exploit the problem structure using ℓ1-norm minimization algorithms.

2.2.1. Optimization problem and voltage sag estimation

The proposed optimization problem is presented in (6). It is often known as ℓ_1-regularized least-squares problem and allows the achievement of the two main goals of this work hypothesis: a sparse vector while the square error is minimized [12]. This approach is widely applied in compressive sensing that is an emerging research area with many applications in the last years.

$$minimize \quad \lambda \cdot \|\mathbf{x}\|_1 + ||\mathbf{A_m} \cdot \mathbf{x} - \mathbf{b}||_2^2 \qquad (6)$$

An optimal state vector $\mathbf{x}^*_{sparse}$ is obtained and the number of voltage sags at *nm* unmetered busbars ($\mathbf{b_{nm}}$), like to SVD, is calculated using equation (7).

$$\mathbf{b_{nm}} = \mathbf{A_{nm}} \cdot \mathbf{x}^*_{sparse} \qquad (7)$$

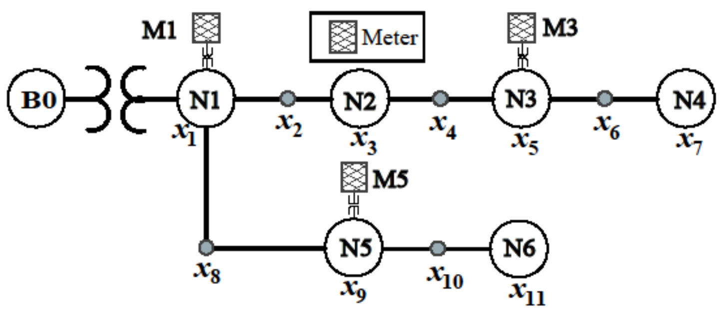


Figure 1. Illustrative three-phase distribution system. **Source:** Own elaboration.

Only a voltage sag detection threshold equal to 0.9 pu was considered in the case study presented in section 3.

2.2.2. Illustrative distribution system

A small distribution system is shown in Figure 1. There are six load nodes, three voltage meters installed and the fifteen points indicate the fault positions defined in the system. **A**$_\mathbf{m}$ and **A**$_\mathbf{nm}$ are built using a detection threshold equal to 0.9 p.u and are shown in Tables 2-3.

In this case study, a set of 40 faults is simulated during a year. Residual voltages are taken from the meters installed and the measurement vector **b** is obtained. Finally, equation (8) is solved and the optimal value **x*** is used in (7) to calculate the voltage sags number at unmetered busbars.

Table 2. **A**$_\mathbf{m}$ for the distribution system in Figure 1.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
M1 (x_1)	1	1	1	0	0	0	0	1	1	0	0
M2 (x_5)	1	1	1	1	1	1	0	1	1	0	0
M3 (x_9)	1	1	1	0	0	0	0	1	1	1	0

Source: Own elaboration.

Table 3. **A**$_\mathbf{nm}$ for the distribution system in Figure 1.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}
(x_2)	1	1	1	1	0	0	0	1	1	0	0
(x_3)	1	1	1	1	1	0	0	1	1	0	0
(x_4)	1	1	1	1	1	0	0	1	1	0	0
(x_6)	1	1	1	1	1	1	1	1	1	0	0
(x_7)	1	1	1	1	1	1	1	1	1	0	0
(x_8)	1	1	1	0	0	0	0	1	1	1	0
(x_{10})	1	1	1	0	0	0	0	1	1	1	1
(x_{11})	1	1	1	0	0	0	0	1	1	1	1

Source: Own elaboration.

$$minimize \quad \lambda \cdot \|\mathbf{x}\|_1 + \left\| \mathbf{Am} \cdot \mathbf{x} - \begin{bmatrix} 25 \\ 35 \\ 12 \end{bmatrix} \right\|_2^2 \qquad (8)$$

2.2.3. ℓ_1-norm minimization algorithms

Two of the most robust and popular solvers of ℓ_1-regularized least-squares problem are presented below.

Gradient Projection for Sparse Reconstruction (GPSR) [16]

GPSR is based on gradient projection algorithms to solve a bound-constrained quadratic programming problem. According to the authors, this approach provides faster solutions compared with interior-point techniques. The model used is presented in (9).

$$minimize \quad \lambda \cdot \|\mathbf{x}\|_1 + \frac{1}{2} ||\mathbf{Ax} - \mathbf{b}||_2^2 \qquad (9)$$

CVX solver [17], [18]

CVX is a robust modeling system to solve disciplined convex programs (DCPs). For the optimization problem shown in (6), CVX exploits the problem structure of a problem that can be reformulated as a convex quadratic program with bound constraints. SDPT3 is an algorithm based on the primal-dual interior-point method that uses the path-following paradigm. Equation (10) presents the model that CVX solves.

$$minimize \quad \lambda \cdot \|\mathbf{x}\|_1 + ||\mathbf{A} \cdot \mathbf{x} - \mathbf{b}||_2^2 \qquad (10)$$

3. Simulation and discuss

The 33-node test distribution system, 12.66 kV, is used to validate the proposed VSE method using ℓ_1-norm minimization algorithms. In other studies of techniques for optimal placement of PMUs are validated using this test system [19]. Also, this electrical system is used in reconfiguration studies, but we will only use its reconfiguration base case shown in figure 2.

The case of study in this paper consists to realize a large number of network faults on the 33-node test distribution system and to apply the VSE proposed to estimate the number of voltage sags at unmetered busbars.

The simulation process is summarized below.

- Stage 1: Zero impedance faults are made in all fault positions using ATP-EMTP package. 128 fault positions are defined in our test system, applying a segmentation of 25 percent in each line. All network fault types are considered: line-to-ground (LG); line-to-line (LL); line-to-line-to-ground (LLG) and three-line (LLL). Three Monte Carlo simulations are performed to validate the proposed VSE method.
- Stage 2: A monitoring system is defined. Nine meters are located according to the methodology presented in [20]. Measurement matrices (**Am** and **Anm**) are built, one for each fault type.

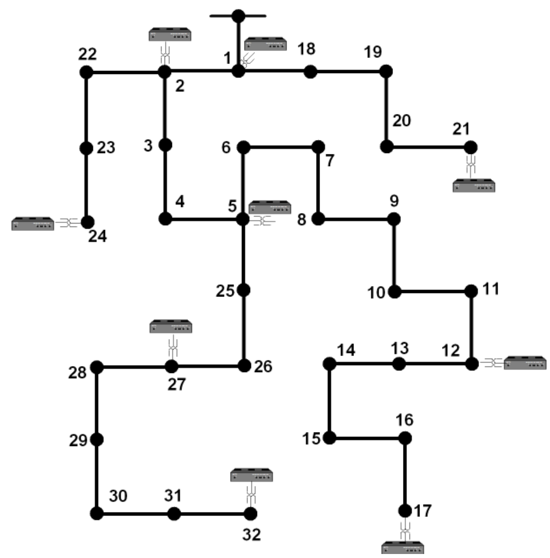


Figure 2. 33-node test distribution system. **Source:** Own elaboration.

The information obtained from the simulation process is equivalent to that provided by the monitoring systems in the real distribution systems. The next step is to apply the VSE proposed. The stages are:

- Stage 1: Measurement vector (**b**) is built from Monte Carlo simulation. A threshold equal to 0.9 pu is considered for ℓ_1-norm methods. Four thresholds, 0.9, 0.7, 0.5 and 0.3 pu, are used in SVD method.
- Stage 2: The optimization problems shown in (2), (8) and (9) are solved. Number of voltage sags at *nm* unmetered busbars (**bnm**) is obtained using (7).

3.1. Validation sceneries obtained from Monte Carlo simulations

Table 4 shows the statistical data used in Monte Carlo scenarios (MC1, MC2 and MC3) [21]. Fault impedances (Z_f) are calculated from a normal distribution.

3.2. Regularization parameter tuning

Optimization problems into of the ℓ_1-minimization solvers are dependent on λ (lambda) parameter, which should be analyzed initially.

Table 4. Statistical data used in Monte Carlo simulations.

MC	Faults	Z_f	Fault type	Total
1	Busbars 0.08 faults/ year Lines 3.7 faults/ year	μ= 5 Ω σ= 2 Ω	LG= 80% LLG= 10% LL= 5% LLL= 5% Randomly locations	1 year: **123** faults
2		μ= 0.5 Ω σ= 0.3 Ω		1 year: **123** faults
3		μ= 5 Ω σ= 2 Ω		20 years **2374** faults

Source: Own elaboration.

A subset of network faults is taken from the validation scenarios and the objective function value is assessed for all fault types. Figure 3 shows the regularization parameter effect on the objective function.

Lambda values lower to 10^{-3} are suitable to achieve the highest accuracy in VSE problem. We take $\lambda=4\times10^{-5}$ in the results presented at the next section.

3.3. Results and discussion

Figure 4 shows the voltage sags number that was incorrectly estimated. The real values of the number of voltage sags are obtained from the simulation process in ATP-EMTP. All 23 unmetered busbars are analyzed for the MC1 and MC2 validation scenarios. Also, all fault types are considered in these results.

Figure 4a shows a typical scenario for one year, where the SVD method presents the lower performance. This is due to its poor performance when network faults with fault impedance are presented. It is generally found that CVX and GPSR solvers present the same accuracy.

Figure 4b aims to show a better performance in SVD method when the network faults have Z_f lower values. Voltage sags number incorrectly estimated is similar in almost all busbars and thus ratifies the hypothesis presented initially. The voltage sags are even better estimated at 8 and 18 busbars using SVD.

Finally, a big set of network faults is assessed. Figure 5 shows the overall performance of the proposed method, where the best global efficiencies are obtained with the ℓ_1-norm methods. It´s clear that this last scenario is not sparse due to all fault positions are faulted. Despite this, the ℓ_1-norm methods still present the best results. The main property of ℓ_1-norm methods is its immunity to high fault impedances, which is the most real conditions in power systems.

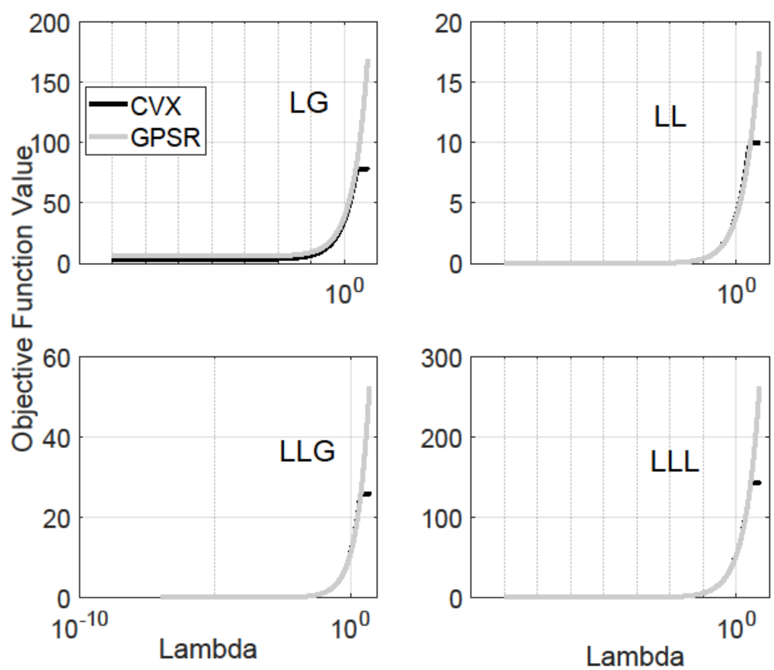
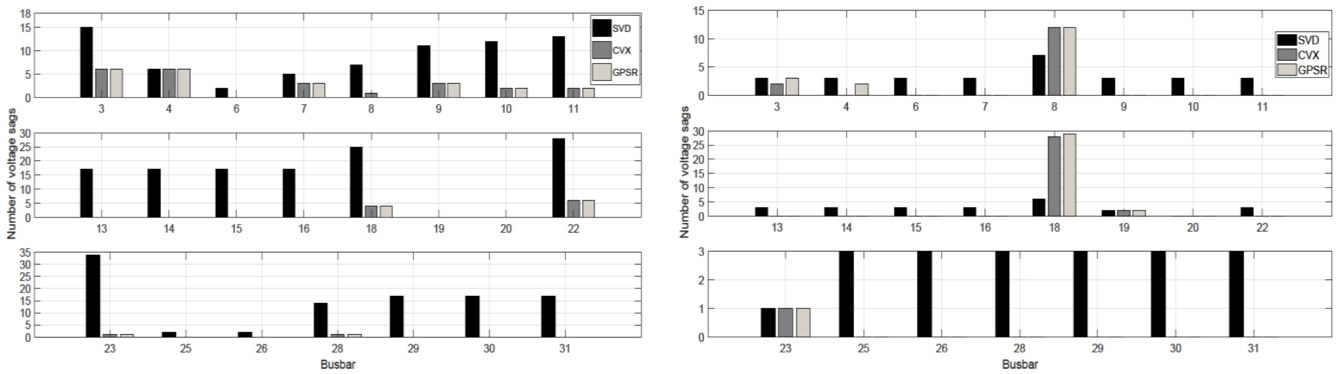


Figure 3. λ parameter versus objective function. **Source:** Own elaboration.



a) b)

Figure 4. Voltage sags number incorrectly estimated. a) MC1; b) MC2 **Source:** Own elaboration.

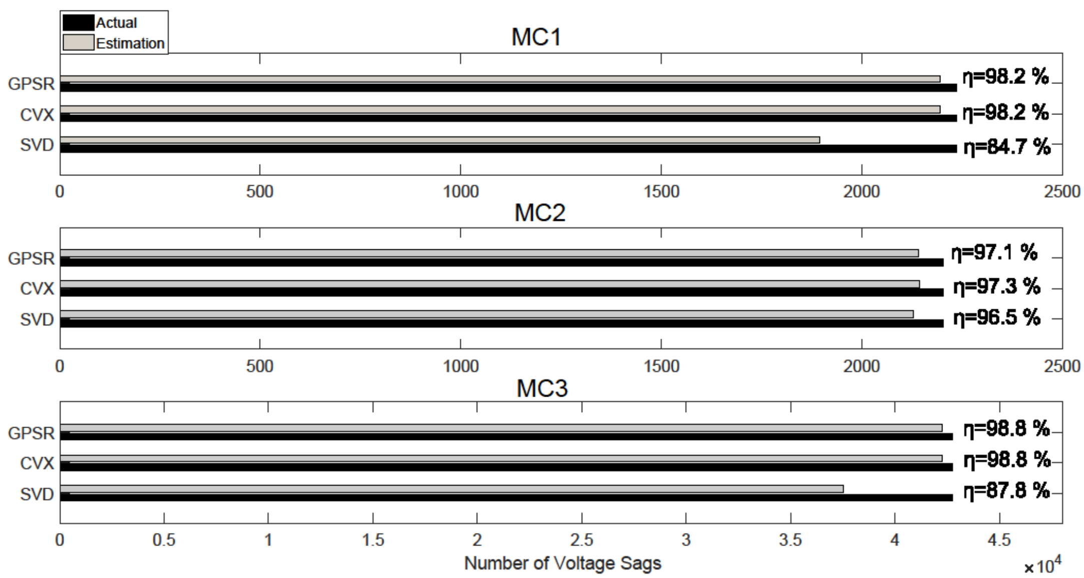


Figure 5. Global efficiencies of the VSE methods. **Source:** Own elaboration.

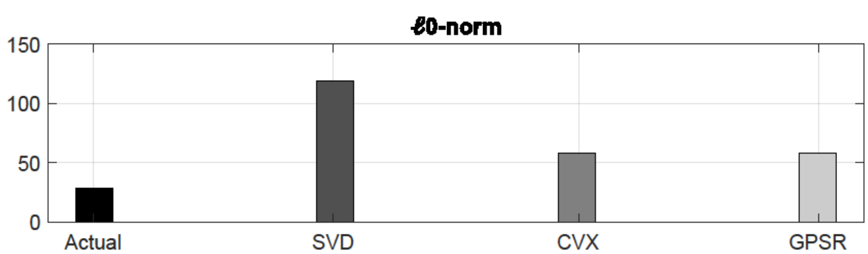


Figure 6. Sparsity property of the optimal vector $\mathbf{x}^*$ in MC1. **Source:** Own elaboration.

These results indicate that the VSE proposed has a good performance under the real conditions of the fault in the radial electric power distribution systems. This fact is decisive for a real implementation.

Figure 6 presents the sparsity of the optimal vector $\mathbf{x}^*$. Line-to-ground faults are only considered because are the most numerous in the validation sceneries. For this, the ℓ_0-norm is calculated and it's compared to the real fault location vector. The MC1 case is only presented because the sparsity is almost null in the other cases. Despite the voltage sags estimation has a good performance, a new mathematical formulation could be proposed where the sparse property will be exploited even more.

These additional results also validate the fact that the problem modelling based on ℓ_1-norm method is coherent with the physical features of the fault networks.
To conclude, several additional issues are discussed for future research:

- Measurement errors were not considered. However, the square error term in (6) allows to consider this issue. The sensitivity to measurement error can be tested in future works.
- The number of fault positions is a criterion to analyze. Typical distances used in fault location programs in distribution systems may be taken into consideration.
- Meters placement is an important issue because it defines the measurement matrix properties [22]. A rigorous study should determine the impact of these properties on the ℓ_1-norm algorithm's performance.

4. Conclusions

Mathematical formulations to voltage sag estimation are very few yet. This work demonstrated that the SVD method presents inaccurate estimations in real fault sceneries with high fault impedances. This paper also proposed a new method for estimating the voltage sags number using a formulation based on the ℓ_1-norm minimization algorithms. Case of studies have shown that ℓ_1-norm methods present the best performance. All fault types and a wide range of fault impedances were considered. The main finding to highlight is the low impact of these variables on the algorithm's accuracy. About computing time and accuracy, the CVX solver worked better in all cases. Finally, the authors highlight new research goals to meet and are already working on new and best mathematical formulations for voltage sags estimation using the compressive sensing theory.

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